A study on rotational motion control for ship steering motion control

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Abstract: In general, a series of ship steering motions is composed of a combination of translational motions and rotational motions of the ship. In particular, a series of rotational motions frequently occurs in narrow areas such as ports and canal zones. In this paper, a method was suggested for composing an integrated control algorithm based on the jog dial as a command instrument for rotational motion control. In order to realize the rotational motions, several algorithms were suggested for generating rotational commands, for selecting motion variables, for choosing reference input values for the motion variables, for computing required accelerations and thrusts, and for allocating thrusts to actuators. A simulation program was compiled to execute simulations for three rotational motions. Finally, the effectiveness of the suggested method was verified by analyzing the simulation results.

Keywords: Ship steering motion, Integrated control algorithm, Rotational motion, Reference input values, Motional variables

1. Introduction

The purpose of ship motion control is largely classified into two problems. One is the problem of ship safety against stranding, submersion, and collision occurring during navigation. The other is the problem of accomplishing a given steering command corresponding to navigating a situation accurately.

In order to solve the above problems, much recent research is proceeding to develop control systems for advanced autopilot, track-keeping control that follows a given route in an electronic chart, automatic mooring, dynamic positioning, and so on [1]-[4].

In particular, high-level technologies for ship steering control must be developed as the traffic in a port is busy and as large-scale ships are emerging. An active steering control technology for automatic mooring in narrow ports is a representative example of the systems necessary to catch up with demands [5]-[9].

In order to realize active steering control, implementation of a type of integrated control system is necessary to generate a series of steering motions under the assumption that the considered ship should be fully actuated [10]-[12].

In this paper, an integrated control system based on a joystick and a jog dial with a command instrument is introduced. Joystick commands from an operator are used for translational steering motions, while jog dial commands are used for rotational commands. Control methods for translational motion in an integrated control system for a fully actuated ship have already been proposed in [13]-[15].

Therefore, a control method for rotational motion in an integrated control system is suggested in this paper. Rotational motions of ships include three types of movement: rotation about an imaginary fore axis, rotation about the center of gravity axis, and rotation about an imaginary aft axis. In order to realize these three rotational motions and to execute the corresponding control command, various algorithms necessary to an integrated control system are suggested. As a method for verifying the suggested algorithms, a simulator based on the MATLAB language is composed. Through many simulations covering several command scenarios, the qualitative effectiveness of the suggested algorithms is proved.

2. Integrated control system for rotation

The structure of an integrated control system that is suggested in this paper for ship rotational motion control based on jog dial commands is depicted in **Figure 1**. The basic structure is very similar to that of the ship translational motion control system suggested in [14].

In order for a ship to carry out a desired rotational motion, for example, a rotation about an imaginary fore axis, two thrust forces along two axes and a thrust moment about one axis must be calculated for the given rotational command, under the assumption that the described coordinate system is a body-fixed coordinate system attached to the ship. Then, these forces must be allocated to actuators installed on the ship. All

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motion variables necessary for control must be measured, fed back, and used to generate a new control command until the given rotational motion command is completely realized. The integrated control system for rotational motion suggested in **Figure 1** contains all algorithms required to realize the given rotational motion command.
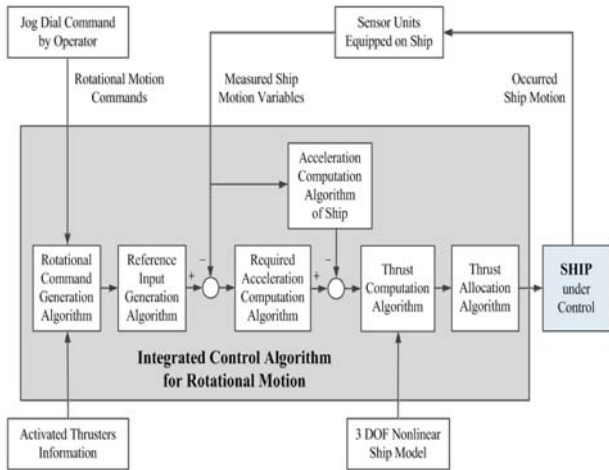

Figure 1: Functional diagram of an integrated control system for ship rotational motion control based on jog dial command

The ship model adopted for the integrated control system is assumed to be as shown in **Figure 2**, and has two translational motions-one each in the x_b axis (surge axis) and the y_b axis (sway axis), and one rotational motion, about the z_b axis (heave axis), under the body-fixed frame of reference: $o - x_b y_b z_b$. In this case, the origin is assumed to be located at the center of gravity. The ship model is fully actuated to generate all required motions without any constraints. There are two controllable-pitch propellers (CPPs) for propulsion, and two side thrusters in both the fore and aft ends for steering.
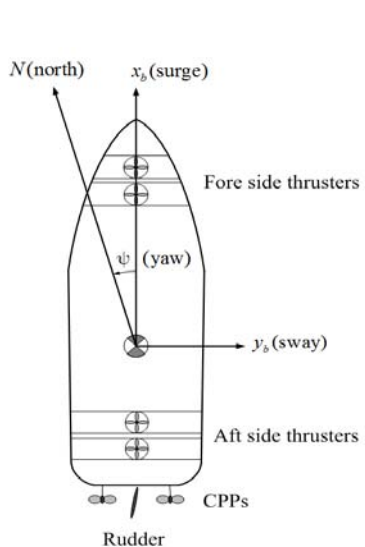

Figure 2: A fully actuated ship model considered

In order to achieve the integrated control system shown in **Figure 1**, the following algorithms are developed. First, it is necessary to develop an algorithm that generates the rotational command given by the operator, and defines the appropriate motion variables corresponding to the command. Next, it is necessary to develop an algorithm that calculates the reference inputs corresponding to the motion variables.

Third, an algorithm to calculate the two required linear accelerations and one angular acceleration is necessary, by comparing the current values of the ship motion variables with reference input values. Fourth, an algorithm to calculate the two required thrust forces and one thrust moment is necessary, by using the required accelerations calculated in the third stage. Lastly, a thrust allocation algorithm is necessary to allocate the required thrust forces and moment to the appropriate actuators.

3. Development of Necessary Algorithms

3.1 Rotation Command, motion variables, and the corresponding reference inputs

3.1.1 Rotation about the axis through a fore center

Consider the rotation of the ship about an imaginary rotation axis through an arbitrary point in the fore side. In order to rotate the ship about this axis, only the two stern thrusters are actuated. In this case, as the fore side point through which the rotation axis passes cannot be fixed, the point moves with the surge velocity and sway velocity in view of the body-fixed coordinate frame. This rotational motion mechanism can be explained as the combination of the translational motion of the center of gravity and the rotational motion about the axis passing through the center of gravity [16], as described in **Figure 3**.
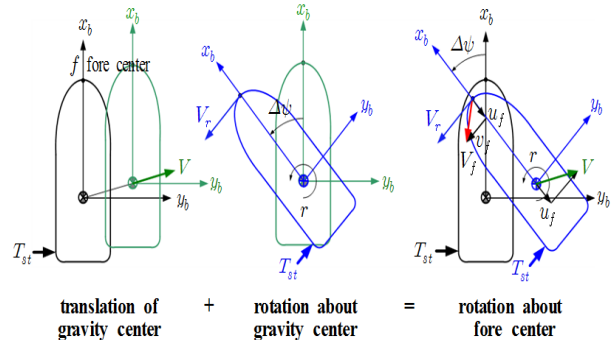

Figure 3: Description of rotational motion about the axis passing through a fore center

Table 1 describes the variables and coordinates used in **Figure 3**. Although only the aft side thrusters with T_{st} are actuated to rotate the ship about the imaginary axis passing

through the fore center f, the ship not only moves with a linear velocity V but also rotates about the axis passing through the center of gravity. The rotation causes a tangential velocity V_r of the fore center f.

Table 1: Description of variables used in **Figure 3**

Variables	Descriptions	Variables	Descriptions
x_b	Surge coordinate	u_f	x_b component of V_f
y_b	Sway coordinate	v_f	y_b component of V_f
V	Velocity of gravity center	r	Angular velocity about gravity center
V_f	Resultant velocity of fore center f	$\Delta\psi$	Increment of yaw angle
V_r	Tangential velocity of f about gravity center	T_{st}	Total thrust by aft side thrusters

As a result, the fore center moves with a velocity V_f, where V_f is the vector sum of u_f and v_f. u_f is the x_b component of the linear velocity V_f, and is equal to the x_b component u of V, because the ship is a rigid body. The y_b component of V_f, that is v_f, is the vector sum of V and V_r in the direction of y_b.

In order that the fore side center f be fixed, the instantaneous linear velocity V_f must be zero. This means that other actuators, such as the CPPs and the fore side thrusters, must be actuated as soon as V_f occurs. From this point of view, the motion variables for command generation and the corresponding reference input values are defined in **Table 2**. x_{fr} stands for the distance from the center of gravity to the fore center.

Table 2: Motion variables and the command reference inputs for rotation about the axis through fore center

Motion variables	Reference input values	Computation of V_f
u_{ref}	$u_{ref} = u_f = 0$	$u_f = u$
v_{ref}	$v_{ref} = v_f = 0$	$v_f = v + x_{fr}\, r$ (+ direction of r : CW)
ψ_{ref}	$\psi_{ref} = \psi_0 + \psi_{JOG}$	ψ_{ref} : Reference yaw angle ψ_0 : Initial yaw angle ψ_{JOG} : Jog dial command angle

3.1.2 Rotation about the axis through an aft center

Consider the rotation of the ship about an imaginary rotation axis through an arbitrary point in the aft side. In order to rotate the ship about this axis, at first, only the two fore thrusters are actuated. In this case, as the aft side point through which the rotation axis passes cannot be fixed, the point moves with the surge velocity and the sway velocity.

This rotational motion mechanism can also be explained as the combination of the translational motion of the center of gravity and the rotational motion about the axis passing through the center of gravity caused by the two fore side thrusters, as described in **Figure 4**.

Table 3 describes the variables and coordinates used in **Figure 4**. Although only the fore side thrusters with T_{fr} are actuated to rotate the ship about the imaginary axis passing through the aft center f, the ship not only moves with a linear velocity V but also rotates about the axis passing through the center of gravity. The rotation causes a tangential velocity V_r of the aft center s.
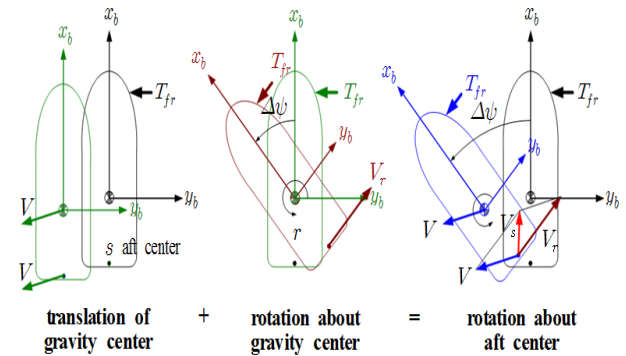

Figure 4: Description of rotational motion about the axis passing through an aft center

Table 3: Description of variables used in **Figure 4**

Variables	Descriptions	Variables	Descriptions
x_b	Surge coordinate	u_s	x_b component of V_s
y_b	Sway coordinate	v_s	y_b component of V_s
V	Velocity of gravity center	r	Angular velocity about gravity center
V_s	Resultant velocity of fore center s	$\Delta\psi$	Increment of yaw angle
V_r	Tangential velocity of s about gravity center	T_{fr}	Total thrust by fore side thrusters

As a result, the fore center moves with a velocity V_s, which is the vector sum of u_s and v_s. u_s is the x_b component of the linear velocity V_s, and is equal to the x_b component u of V, because the ship is a rigid body. The y_b component of V_s, that is v_s, is the vector sum of V and V_r in the direction of y_b.

In order that the aft side center s be fixed, the instantaneous linear velocity V_s must be zero. This means that other actuators, such as the CPPs and the aft side thrusters, must be actuated as soon as V_s occurs. From this perspective, the motion variables for command generation and the corresponding reference input values for rotation about the axis passing

through the aft center are defined in **Table 4**. x_{st} represents the distance from the center of gravity to the aft center.

Table 4: Motion variables and the command reference inputs for rotation about the axis through aft center

Motion variables	Reference input values	Computation of V_s
u_{ref}	$u_{ref} = u_s = 0$	$u_s = u$
v_{ref}	$v_{ref} = v_s = 0$	$v_s = v + x_{st}\, r$ (+ direction of r : CW)
ψ_{ref}	$\psi_{ref} = \psi_0 + \psi_{JOG}$	ψ_{ref} : Reference yaw angle ψ_0 : Initial yaw angle ψ_{JOG} : Jog dial command angle

3.1.3 Rotation about the axis through a gravity center

Consider the rotation of the ship about an imaginary rotation axis passing through the center of gravity. In order to rotate the ship about this axis, the two fore thrusters and two aft thrusters must be actuated simultaneously. In the case in which the moment generated by the fore side thrusters is the same as that of the aft side thrusters, the center of gravity does not move, but behaves as if it were fixed.

This rotational motion mechanism can be explained as the combination of the motions of the two cases already discussed. **Figure 5** demonstrates the rotational motion mechanism about the rotation axis passing through the center of gravity. Nevertheless, the linear velocity of the center of gravity may not be equal to zero, because the thrust forces by the fore side thrusters and the aft side thrusters are opposite, but their magnitudes are slightly different from each other.

Table 5 describes variables used in **Figure 5**.

Table 6 represents the motion variables for command generation, as well as the corresponding reference input values, for rotation about the axis passing through the center of gravity, in order for the velocity of the center of gravity to be zero by way of control.
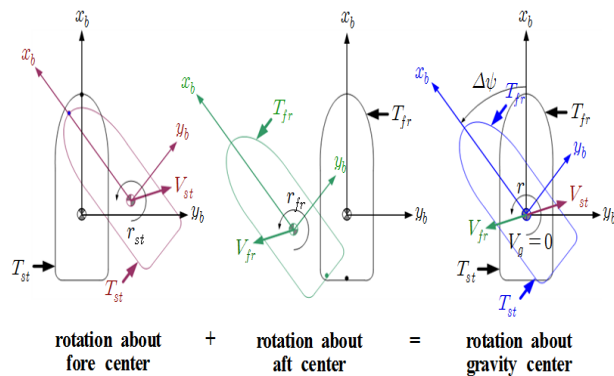

Figure 5: Description of rotational motion about the axis passing through a gravity center

Table 5: Description of variables used in **Figure 5**

Variables	Description	Variables	Description
T_{st}	Aft thruster force	T_{fr}	Fore thruster force
V_{st}	Linear velocity of gravity center by T_{st}	V_{fr}	Linear velocity of gravity center by T_{fr}
r_{st}	Angular velocity about gravity center axis by T_{st}	r_{fr}	Angular velocity about gravity center axis by T_{fr}
V	Linear velocity of gravity center	$\Delta\psi$	Yaw angle increment
r	Total angular velocity $r = r_{st} + r_{fr}$	u	x_b component of V measured
v	y_b component of V measured	-	-

Table 6: Motion variables and the command reference inputs for rotation about the axis through gravity center

Motion variables	Reference input values	Descriptions
u_{ref}	$u_{ref} = u = 0$	Reference surge velocity
v_{ref}	$v_{ref} = v = 0$	Reference sway velocity
ψ_{ref}	$\psi_{ref} = \psi_0 + \psi_{JOG}$	ψ_{ref} : Reference yaw angle ψ_0 : Initial yaw angle ψ_{JOG} : Jog dial command angle

3.2 Computation of required accelerations

The required linear accelerations in the directions of the x_b and y_b axes, as well as the required angular acceleration about the z_b axis, are computed by comparing the reference values with the current motion values measured at the ship at each sampling time. Although accelerations vary continuously, it is possible to suppose that they are constant during the chosen sampling interval. Under this assumption, algorithms to compute the average accelerations are suggested.

At the current time nT_s, it is supposed that the reference velocity in the x_b direction is given as u_{ref}, and the velocity of the ship in the x_b direction is measured as $u(n)$. The average linear acceleration in the x_b direction is computed by **Equation (1)**, where T_s is the sampling period:

$$\dot{u}(n) = \frac{u_{ref} - u(n)}{T_s} \tag{1}$$

When the reference velocity in the y_b direction is given as v_{ref} and the velocity of the ship in the y_b direction is measured as $v(n)$, the average linear acceleration in the y_b direction is computed by **Equation (2)**:

$$\dot{v}(n) = \frac{v_{ref} - v(n)}{T_s} \tag{2}$$

From these equations, it is clear that the reference angular velocity and the current angular velocity must be computed prior

to computing the angular acceleration. They can be computed using **Equation (3)** and **(4)**, where ψ_{ref} is the given reference yaw angle, $\psi(n)$ is the current yaw angle measured at the ship, $r_{ref}(n)$ is the computed reference angular velocity, and $r_s(n)$ is the computed current angular velocity about the z_b axis.

$$r_{ref}(n) = \frac{\psi_{ref} - \psi(n)}{T_s} \qquad (3)$$

$$r_s(n) = \frac{\psi(n) - \psi(n-1)}{T_s} \qquad (4)$$

From **Equation (3)** and **(4)**, the required average angular acceleration $\dot{r}(n)$ about the z_b axis is computed by **Equation (5)**:

$$\dot{r}(n) = \frac{r_{ref}(n) - r_s(n)}{T_s} \qquad (5)$$

Equation (5) is used whenever $\psi(n)$ is not equal to ψ_{ref} at the current sampling time nT_s. Incidentally, if $\psi(n)$ is equal to ψ_{ref}, **Equation (6)** and **(7)** are used to compute $\dot{r}(n)$, whether $r_s(n)$ is equal to zero or not.

$$\dot{r}(n) = 0, \quad \text{for} \quad r_s(n) = 0 \qquad (6)$$

$$\dot{r}(n) = \frac{0 - r_s(n)}{T_s}, \quad \text{for} \quad r_s(n) \neq 0 \qquad (7)$$

Figure 6 presents the flowchart of the algorithms for computing the required average linear accelerations in the direction of the x_b and y_b axes, and for computing the required average angular acceleration about the z_b axis.
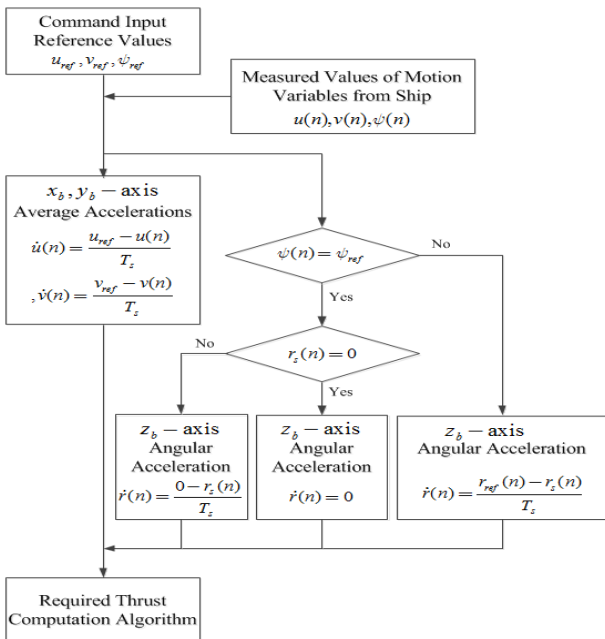

Figure 6: Algorithms for computing the required linear accelerations and angular acceleration

3.3 Computation of required thrust forces and thrust moment

In order for a ship to acquire the motion commanded by the operator, the required thrust forces in the direction of the x_b and y_b axes, and the moment about the z_b axis must be computed. To do this, the rigid body dynamics for the ship itself, as well as the hydrodynamics of the water surrounding the ship, must be considered. The rigid body dynamics of the main elements to be considered include the inertial forces and moments of the ship. Hydrodynamics from the motion of the surrounding water includes additional inertial forces and moments, added damping forces and moments, restoring forces and moments, etc.

In this paper, the ship motion under consideration is assumed to be a low-speed steering motion, and is thus assumed to be described by nonlinear three degrees-of-freedom differential equations in the surge, sway, and yaw directions. Therefore, the following assumptions are made about the controlled ship and its motion **[2][8]**:

(1) The considered ship is symmetric with reference to the $o - x_b z_b$ plane, and the origin of the body-fixed frame is at the center of gravity.

(2) The ship exhibits planar motion in the $o - x_b y_b$ plane. Thus, restoring forces and moments are neglected from the viewpoint of hydrostatics. In addition, only the three axial motions in the surge, sway, and yaw directions are coupled.

(3) As the ship is under a low-speed steering motion, the second-order nonlinear damping effects due to high-speed motion are neglected.

Under the above assumptions, the equations of motion for the ship are approximately represented as **Equation (8) - (10) [14]**.

$$(m - X_{\dot{u}})\dot{u} = (m - Y_{\dot{v}})vr + X_C \qquad (8)$$

$$(m - Y_{\dot{v}})\dot{v} = Y_v v + (Y_r - mu)r + Y_\delta \delta + Y_C \qquad (9)$$

$$(I_z - N_{\dot{r}})\dot{r} = N_v v + N_r r + N_\delta \delta + N_C \qquad (10)$$

where m is the mass of the ship, I_z is the moment of inertia about the z_b axis, $-X_{\dot{u}}, -Y_{\dot{v}},$ and $-N_{\dot{r}}$ are the added masses and moment, respectively, $Y_v, Y_r, Y_\delta, N_v, N_r,$ and N_δ are the hydrodynamic derivatives, X_C, Y_C are thrust forces exerted by the actuators, and N_C is the thrust moment exerted by the actuators.

The reason for describing the equations of motion is to derive recursive equations for computing the required thrust forces and moment so that the control system may generate

the rotation command. Therefore, **Equation (8) - (10)** are re-arranged as the recursive **Equation (11) - (13)** about the thrust forces and moment for X_C, Y_C, and N_C at sampling time $t = (n+1)T_s$.

$$X_C(n+1) = (m - X_{\dot{u}})[\dot{u}(n) - \dot{u}_s(n)] \\ - (m - Y_{\dot{v}})v(n)r(n) \tag{11}$$

$$Y_C(n+1) = (m - Y_{\dot{v}})[\dot{v}(n) - \dot{v}_s(n)] - Y_v v(n) \\ - [Y_r - m\,u(n)]r(n) - Y_\delta \delta(n) \tag{12}$$

$$N_C(n+1) = (I_z - N_{\dot{r}})[\dot{r}(n) - \dot{r}_s(n)] - N_v v(n) \\ - N_r r(n) - N_\delta \delta(n) \tag{13}$$

where $u(n)$, $v(n)$, and $r_s(n)$ are the current linear and angular velocities, $\delta(n)$ is the rudder angle at the current time nT_s, $\dot{u}(n)$, $\dot{v}(n)$, and $\dot{r}(n)$ are the required linear and angular accelerations that must be utilized at nT_s in order to satisfy the rotation command, and $\dot{u}_s(n)$, $\dot{v}_s(n)$, and $\dot{r}_s(n)$ are the current linear and angular accelerations of the ship.

Therefore, the required thrust forces and moment at the next sampling time $t = (n+1)T_s$ are obtained from **Equation (11) - (13)** by using only the previously measured information about the motion of the ship at the current sampling time $t = nT_s$.

3.4 Thrust allocation algorithm

In order to correctly allocate the required forces and moment to the actuators installed on the ship, appropriate and optimal allocation algorithms are suggested.

In this paper, the considered ship is assumed to be the fully actuated ship shown in **Figure 7**.
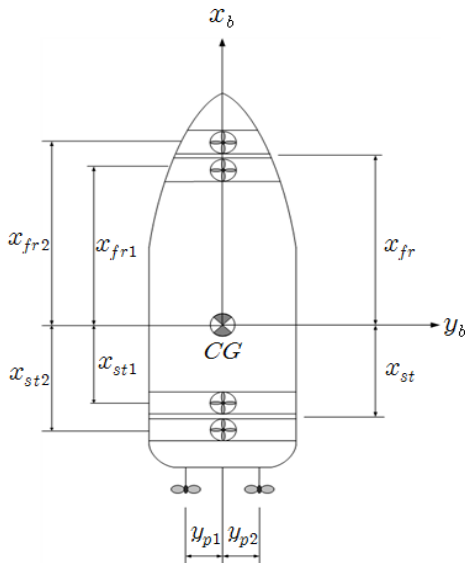

Figure 7: Fully actuated ship and installed actuator positions considered in this paper

Information about the actuator positions installed on the ship shown in **Figure 7** is given in **Table 7**.

Table 7: Description of installed positions of actuators

Positions	Descriptions	Positions	Descriptions
y_{p1}	Perpendicular distance from x_b to CPP 1	x_{fr2}	Distance from CG to fore side thrusters 2
y_{p2}	Perpendicular distance from x_b to CPP 2	x_{st}	Average distance from CG to aft side thrusters
x_{fr}	Average distance from CG to fore side thrusters	x_{st1}	Distance from CG to aft side thruster 1
x_{fr1}	Distance from CG to fore side thruster 1	x_{st2}	Distance from CG to aft side thruster 2

3.4.1 Allocation algorithm for rotation about the axis through the fore center

The premise conditions and assumptions for allocating the required thrust forces and moments to specific thrusters, so as to accomplish the commanded rotation about the fore center axis, are as follows:

(1) The required thrust force $Y_C(n+1)$ that is computed to force the linear velocity v_f to be equal to zero is allocated to the fore side thrusters and the aft side thrusters at the same time.

(2) The required thrust force $X_C(n+1)$ that is computed to cause the linear velocity u_f to be equal to zero is allocated only to the CPPs.

(3) Although the required thrust moment $N_C(n+1)$ about the z_b axis is allocated to the aft side thrusters, it contains the moment generated by the fore side thrusters to produce a v_f equal to zero.

Under the above assumptions, all the thrust forces of the actuators are computed by **Equation (14) - (19)**:

$$T_{cpp1}(n+1) = \frac{1}{2}X_C(n+1) \tag{14}$$

$$T_{cpp2}(n+1) = \frac{1}{2}X_C(n+1) \tag{15}$$

$$T_{fr1}(n+1) = \frac{(x_{fr} - x_{fr2})}{(x_{fr1} - x_{fr2})}Y_C(n+1) \tag{16}$$

$$T_{fr2}(n+1) = -\frac{(x_{fr} - x_{fr1})}{(x_{fr1} - x_{fr2})}Y_C(n+1) \tag{17}$$

$$T_{st1}(n+1) = a_1 Y_C(n+1) + a_2 N_C(n+1) \tag{18}$$

$$T_{st2}(n+1) = b_1 Y_C(n+1) + b_2 N_C(n+1) \tag{19}$$

The coefficients a_1, a_2, b_1, and b_2 are calculated as follows:

$$a_1 = \frac{x_{fr}(x_{st2} - x_{st})}{x_{st}(x_{st1} - x_{st2})}, \; a_2 = -\frac{(x_{st2} - x_{st})}{x_{st}(x_{st1} - x_{st2})}$$

$$b_1 = -\frac{x_{fr}(x_{st1} - x_{st})}{x_{st}(x_{st1} - x_{st2})}, \; b_2 = \frac{(x_{st1} - x_{st})}{x_{st}(x_{st1} - x_{st2})}$$

3.4.2 Allocation algorithm for rotation about the axis through the aft center

The premise conditions and assumptions for allocating the required thrust forces and moments to specific thrusters, so as to accomplish the commanded rotation about the aft center axis, are as follows:

(1) The required thrust force $Y_C(n+1)$ that is computed to force the linear velocity v_s to be equal to zero is allocated to the fore side thrusters and the aft side thrusters at the same time.

(2) The required thrust force $X_C(n+1)$ that is computed to cause the linear velocity u_s to be equal to zero is allocated only to the CPPs.

(3) Although the required thrust moment $N_C(n+1)$ about the z_b axis is allocated to the fore side thrusters, it contains the moment generated by the aft side thrusters to produce a v_s equal to zero.

Under the above assumptions, all the thrust forces of the actuators are computed by **Equation (20) - (25)**:

$$T_{cpp1}(n+1) = \frac{1}{2}X_C(n+1) \tag{20}$$

$$T_{cpp2}(n+1) = \frac{1}{2}X_C(n+1) \tag{21}$$

$$T_{fr1}(n+1) = c_1 Y_C(n+1) + c_2 N_C(n+1) \tag{22}$$

$$T_{fr2}(n+1) = d_1 Y_C(n+1) + d_2 N_C(n+1) \tag{23}$$

$$T_{st1}(n+1) = -\frac{x_{st2} - x_{st}}{x_{st1} - x_{st2}} Y_C(n+1) \tag{24}$$

$$T_{st2}(n+1) = \frac{x_{st1} - x_{st}}{x_{st1} - x_{st2}} Y_C(n+1) \tag{25}$$

The coefficients c_1, c_2, d_1, and d_2 are calculated as follows:

$$c_1 = \frac{x_{st}(x_{bo2} - x_{bo})}{x_{bo}(x_{bo1} - x_{bo2})}, \; c_2 = -\frac{(x_{bo2} - x_{bo})}{x_{bo}(x_{bo1} - x_{bo2})}$$

$$d_1 = -\frac{x_{st}(x_{bo1} - x_{bo})}{x_{bo}(x_{bo1} - x_{bo2})}, \; d_2 = \frac{(x_{bo1} - x_{bo})}{x_{bo}(x_{bo1} - x_{bo2})}$$

3.4.3 Allocation algorithm for rotation about the axis through the gravity center

The premise conditions and assumptions for allocating the required thrust forces and moments to specific thrusters, so as to accomplish the commanded rotation about the axis passing through the center of gravity, are as follows:

(1) The required thrust moment $N_C(n+1)$ that is computed to rotate the ship about the axis passing through the axis through the center of gravity, is allocated to the fore side thrusters and the aft side thrusters in the manner in which the coupled moments occur.

(2) The required thrust force $X_C(n+1)$ that is computed to force the linear velocity u to be equal to zero is allocated only to the CPPs.

(3) The required thrust force $Y_C(n+1)$ that is computed to cause the linear velocity v to be equal to zero is allocated to the fore side thrusters and the aft side thrusters.

Under the above assumptions, all the allocated thrust forces of the actuators are computed by **Equation (26) - (31)**.

$$T_{cpp1}(n+1) = \frac{1}{2}X_C(n+1) \tag{26}$$

$$T_{cpp2}(n+1) = \frac{1}{2}X_C(n+1) \tag{27}$$

$$T_{fr1}(n+1) = e_1 Y_C(n+1) + e_2 N_C(n+1) \tag{28}$$

$$T_{fr2}(n+1) = g_1 Y_C(n+1) + g_2 N_C(n+1) \tag{29}$$

$$T_{st1}(n+1) = h_1 Y_C(n+1) + h_2 N_C(n+1) \tag{30}$$

$$T_{st2}(n+1) = k_1 Y_C(n+1) + k_2 N_C(n+1) \tag{31}$$

The coefficients e_1, e_2, g_1, g_2, h_1, h_2, k_1, and k_2 are calculated as follows:

$$e_1 = \frac{(-x_{fr} + x_{fr2})x_{st}}{(-x_{fr}x_{fr2} + x_{st}x_{fr2} + x_{fr1}x_{fr} - x_{fr1}x_{st})}$$

$$e_2 = -\frac{-x_{fr} + x_{fr2}}{(-x_{fr}x_{fr2} + x_{st}x_{fr2} + x_{fr1}x_{fr} - x_{fr1}x_{st})}$$

$$g_1 = -\frac{(x_{fr1} - x_{fr})x_{st}}{(-x_{fr}x_{fr2} + x_{st}x_{fr2} + x_{fr1}x_{fr} - x_{fr1}x_{st})}$$

$$g_2 = \frac{x_{fr1} - x_{fr}}{(-x_{fr}x_{fr2} + x_{st}x_{fr2} + x_{fr1}x_{fr} - x_{fr1}x_{st})}$$

$$h_1 = \frac{x_{fr}(-x_{st2} + x_{st})}{x_{st1}x_{fr} - x_{st1}x_{st} - x_{st2}x_{fr} + x_{st2}x_{st}}$$

$$h_2 = \frac{x_{st2} - x_{st}}{x_{st1}x_{fr} - x_{st1}x_{st} - x_{st2}x_{fr} + x_{st2}x_{st}}$$

$$k_1 = -\frac{x_{fr}(-x_{st1}+x_{st})}{x_{st1}x_{fr}-x_{st1}x_{st}-x_{st2}x_{fr}+x_{st2}x_{st}}$$

$$k_2 = \frac{-x_{st1}+x_{st}}{x_{st1}x_{fr}-x_{st1}x_{st}-x_{st2}x_{fr}+x_{st2}x_{st}}$$

4. Simulations and analysis of results

4.1 Specifications of the ship and hydrodynamic derivatives

Several simulations were conducted to verify the effectiveness of the suggested algorithms for the three rotational motions of the ship. The controlled ship model is of the SR-108 container ship [8], which has two side thrusters each at the fore and aft sides, two CPPs, and one rudder. Its specifications are given in **Table 8**, and the hydrodynamic derivatives used in **Equation (11) - (13)** are specified in **Table 9**.

Table 8: Principal dimensions of the controlled ship

Hull		
Length over all	$L_{OA}\,(m)$	188.0
Breadth	$B\,(m)$	25.4
Depth	$D\,(m)$	15.4
Draft	$d\,(m)$	8.5
Trim	$\tau\,(m)$	1.0
Block coefficient	C_B	0.559
Prismatic coefficient	C_p	0.58

Table 9: Hydrodynamic derivatives calculated from the dimensions of the controlled ship

Derivatives	Values	Derivatives	Values
m	220940	Y_r	$1202800 \times V$
I_z	422892968	Y_δ	$-5882 \times V^2$
X_v	11047	N_v	$-1207400 \times V^2$
Y_v	1767356	N_r	$-106110000 \times V$
N_r	39482.1	N_δ	$510730 \times V^2$
Y_v	$-22332 \times V^2$	-	-

The purpose of this paper is to verify the feasibility of the controlled ship attaining the commanded rotational motion when the suggested algorithms are applied. In this paper, therefore, the actuator dynamic models were neglected, and all actuators immediately and accurately took on the required thrusts. Moreover, the rudder was assumed to be an additional fixed structure, whose action was neglected because the rotational motion was a low-speed steering motion.

4.2 Simulations for performance verification

To verify the control performance for rotational motion of the ship, a simulation model was composed based on the MATLAB Simulink software as shown in **Figure 8**.
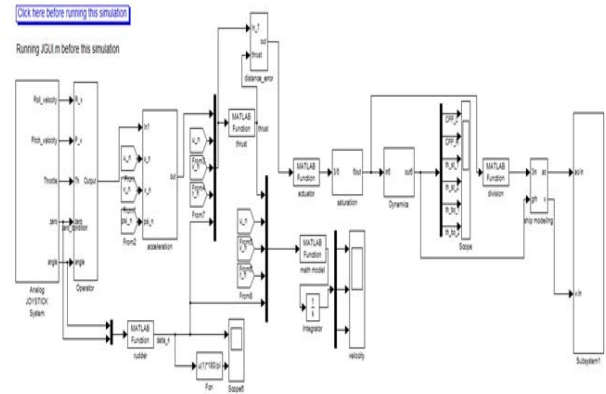

Figure 8: Simulation model for rotational motion of the ship designed using MATLAB Simulink

In **Figure 8**, a number of MATLAB functions are shown, such as a function to identify the rotational command given by the operator, a function to generate the rotational command, a function to generate reference inputs, a function to compute the required accelerations, a function to compute the required thrust forces and moment, a function to allocate the thrusts to the actuators, a function to define the ship model, a function to choose the rudder angular position, etc. Detailed explanations of such functions are abbreviated here, but are fully explained in **[13]**.

4.2.1 Rotation about the axis through the fore center

In order to investigate the behavior of the ship under a rotation command about the axis passing through the fore center, two simulations were carried out by using the simulation program developed in this paper, based on MATLAB's Simulink.

The first simulation was performed to investigate whether the fore center would move or not, by driving only the aft side thrusters with thrust force T_{st}. The initial yaw angle was $\psi(0) = 0[°]$, and the command rotation angle was $\psi_{ref} = 135\ [°]$.

Figure 9 shows the simulation result of the behavior of the ship. The unit of distance on the horizontal and vertical axes was the meter (m). From the figure, it was known that the fore center (denoted by the red line) moved with linear velocities u_f and v_f. As a result, the axis passing through the fore center cannot play the role of a fixed hinge point.

The resultant motion shown in **Figure 9** was explained by investigating the motion variables that occurred for the ship during the simulation. **Figure 10** demonstrates the motion variables at time nT_s, that is, $u_f(n)$, $v_f(n)$, $r(n)$, and $\psi(n)$.
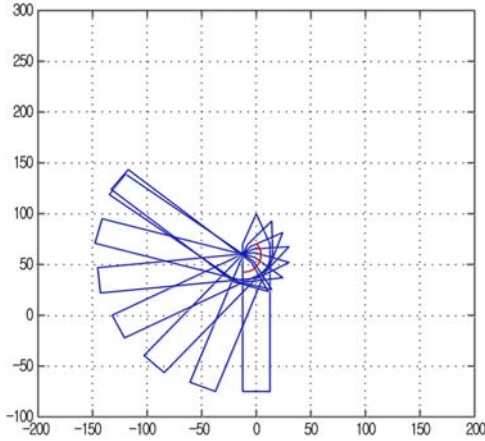
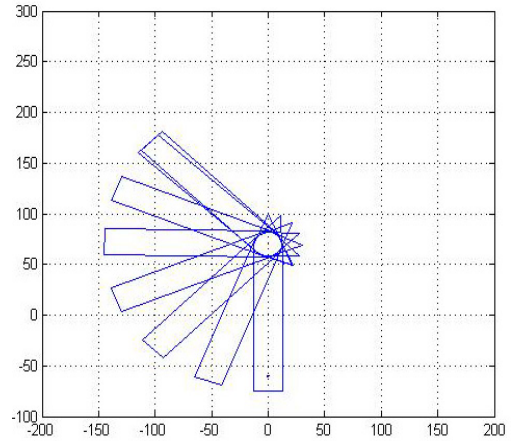**Figure 9:** Result of rotational motion in case that only aft side thrusters were actuated with thruster force T_{st}
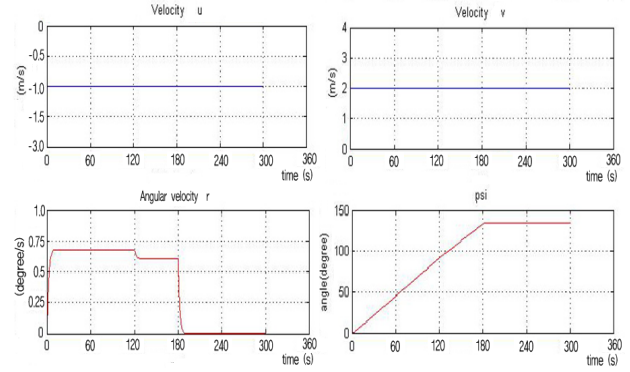

Figure 10: Motion variables corresponding to **Figure 9**

From **Figure 10**, it was seen that the velocities $u_f(n)$ and $v_f(n)$ of the fore center occurred in spite of the rotational command being for the rotation about the axis passing through the fore center.

In order to solve the problem resulting from the first simulation, the integrated control algorithm suggested in this paper was applied to accomplish the rotation about the axis passing through the fore center, in the second simulation. The simulation conditions were the same as for the first simulation.

Figure 11 presents the behavior of the ship for the second simulation, and **Figure 12** displays the ship motion variables that occurred in the ship.

It can be seen from **Figure 11** that the fore center did not move, but acted as if it were a fixed point. Furthermore, it is seen from **Figure 12** that the velocities $u_f(n)$ and $v_f(n)$ were equal to zero. Therefore, the effectiveness of the suggested control algorithm was proven.

It is important to note that the actuator dynamic models, such as those for the CPPs and the side thrusters, were not considered. From this point of view, the simulation results and the effectiveness of the suggested control algorithm must be analyzed in a qualitative manner.

Figure 11: Result of rotational motion in case that the suggested control algorithm is applied

Figure 12: Motion variables corresponding to **Figure 11**

4.2.2 Rotation about the axis through the aft center

In order to investigate the behavior of the ship under a rotation command about the axis passing through the aft center, another simulation was performed. The initial yaw angle was $\psi(0) = 0$ [°], and the command rotation angle was $\psi_{ref} = 270$ [°]. The simulation result referred only to the behavior of the ship for the purpose of proving the possibility of qualitative control, as shown in **Figure 13**.
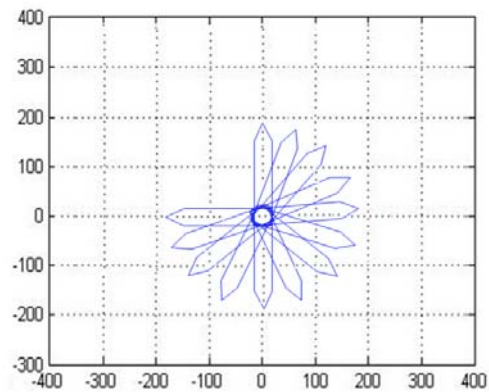

Figure 13: Result of rotational motion about the axis through the gravity center

As was the case for the rotation about the axis through the fore center, the velocities $u_s(n)$ and $v_s(n)$ of the aft center were seen to be equal to zero. This means that the aft center could be considered as an imaginary fixed point.

Therefore, it was convincingly shown that the suggested algorithms for the integrated control system operated well.

4.2.3 Rotation about the axis through the gravity center

In order to investigate the behavior of the ship under a rotation command about the axis passing through the center of gravity, another simulation was effected. The initial yaw angle was $\psi(0) = 0$ [°], and the command rotation angle was $\psi_{ref} = 90$ [°]. The simulation result referred only to the behavior of the ship to prove the qualitative possibility of the suggested algorithms. The simulation result, shown in **Figure 14**, portrayed the behavior of the ship corresponding to the center of gravity rotational command.
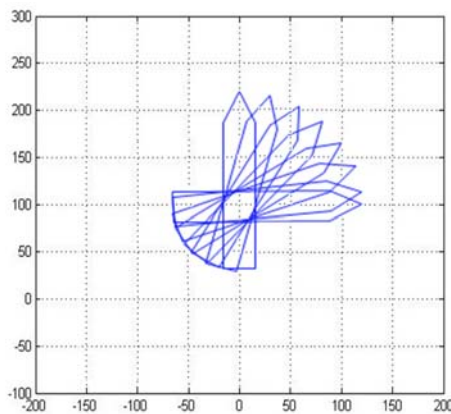

Figure 13: Result of rotational motion about the axis through the gravity center

As for the two cases described previously, the velocities $u_s(n)$ and $v_s(n)$ of the center of gravity were observed to be equal to zero. This means that the center of gravity could be considered as an imaginary fixed point. Therefore, it was confirmed that the suggested algorithms for the rotation about the axis passing through the center of gravity were qualitatively well suited.

5. Conclusions

In this paper, an integrated control system was suggested for the rotational motion control of a ship under a low-speed steering motion. The controlled ship was assumed to be fully actuated in order for the ship to accomplish three specific types of rotational motion without any constraints. The suggested control system was composed of several algorithms, such as an algorithm to generate rotational commands, an al-

gorithm to define motion variables, an algorithm to generate reference input values, an algorithm to compute the required linear and angular accelerations, an algorithm to compute the required thrust forces and moment, and an algorithm to allocate thrust forces and moment to actuators.

In order to verify the effectiveness of several of the suggested algorithms, various simulations were performed based on a simulation program developed using MATLAB. By means of the simulation results, the integrated control algorithm, including several individual algorithms, was assured of being qualitatively effective.

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