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Original Paper

Performance improvement for velocity controller of fish sorting belt conveyor system under bounded disturbance in transient phase

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Abstract: This paper proposes a modified-model reference adaptive controller (M-MRAC) for the velocity control of a belt conveyor in a fish sorting system with uncertainty parameters, input saturation, and bounded disturbances. The following aspects are addressed to improve the tracking performance and robustness of the proposed controller for the fish sorting belt conveyor system with bounded disturbances in the transient phase. First the estimated parameters in adaptive laws have smooth variations under bounded external disturbances and a σ-modification is added to the adaptive law for the proposed M-MRAC controller to be robust. Second, an auxiliary error vector is introduced for compensating the error dynamics of the system when the input saturation occurs. Finally, the experimental results of the proposed controller under bounded disturbance and saturated input are better in effectiveness and performance than those of a conventional model reference adaptive controller.

Keywords: Bounded Disturbance, M-MRAC, σ-modification, Saturated Input, velocity controller

1. Introduction

In a fish sorting system (FSS), captured fishes are transported by a fish pump from a ship into a fish sorting line through a conveyor system. The captured fishes are sorted by the injury rate, which is estimated by using an image processing system. Thus, the conveyor system speed plays a key role in estimating the injury rate of fishes with high accuracy. Accordingly, the velocity of the conveyor system should be controlled with suitable speed to achieve reliable recognition with the image processing system. The conveyor system in a FSS consists of three or more conveyors, the desired velocities of which are defined as trapezoidal velocity profiles **[1]**. To control the closed-loop dynamics of the conveyor system close to the desired velocities, a system model is used to develop a model-based controller. However, the conveyor system has some uncertain parameters, such as a friction factor, belt elastic factor, pulling force, and others, which are unmeasured in the conveyor system.

An adaptive controller was considered for its important ability to deal with system uncertainties without requiring explicit, unknown plant parameter identification **[2][3]**. A conventional model reference adaptive controller (CMRAC) tuning control parameters directly is one of the main schemes utilized in the field of adaptive control **[4]-[6]**. Although asymptotic tracking could be achieved in CMRAC systems, the tracking performance in the transient state could be poor **[7]** because it is impossible to achieve a small deviation of the tracking error in the transient state with an insufficient adaptation rate.

For engineering systems, it should be noted that the control input signal is frequently saturated and has proved to be a source of performance degradation. An input control signal saturation could lead to poor control performance and even closed-loop system instability **[8][9]** if its effect is not considered in the design of the controller.

In addition, the estimated parameters in the update laws can be varied smoothly in the presence of bounded disturbances. Therefore, certain modifications **[10]**, such as the dead-zone technique, e-modification, or projection operator were utilized to the modify update laws.

This paper proposes a model reference adaptive control approach, in which the reference model is modified by a feedback

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of a modeling error signal **[11]**, and applied to the velocity control of the conveyor system in a FSS with uncertain parameters, saturated input, and bounded disturbances. In the presence of bounded disturbance, a σ-modification is utilized in the update laws of the proposed controller to eliminate the drift phenomenon of the control parameters. In addition, an auxiliary error vector **[12]** is employed to compensate for the error dynamics when input saturation occurs. The experimental results verify the effectiveness and the performance of the proposed controller.

2. System Modeling

A typical conveyor system in an FSS, as shown in **Figure 1**, consists of an on-loading conveyor (*1*st conveyor), a camera conveyor (*2*nd conveyor) to examine moving fish boxes using an image processing system, and a transition conveyor (*3*rd conveyor). Each conveyor consists of a mechanical subsystem and an electrical subsystem. The simplified model of the mechanical subsystem of the *i*th conveyor of the FSS is shown in **Figure 2** (i = 1, 2, 3).

In **Figure 2**, J_{i1} and J_{i2} are the moments of inertia of the driving and driven rollers, ω_{i1} and ω_{i2} are the angular velocities of the driving and driven roller, f_{i1} and f_{i2} are friction coefficients of bearings inside the driving and driven rollers, and D_{i1} and D_{i2} are diameters of the driving and driven rollers, respectively.

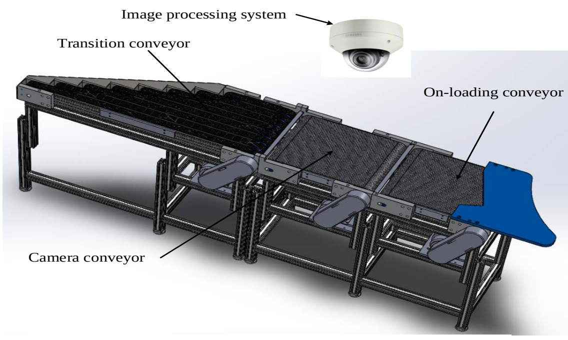

Figure 1: Typical conveyor system in a FSS

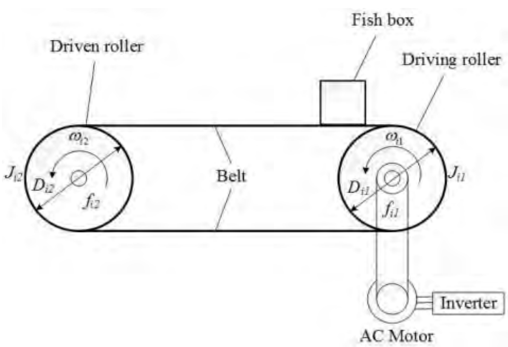

Figure 2: Simplified model of the *i*th conveyor of FSS

To simplify the mechanical subsystem modeling, **Assumption 1** is proposed as follows:

- **Assumption 1**
  - ✓ The connection between the motor shaft and driving roller is rigid and short.
  - ✓ Belt slippage on the rollers is negligible.
  - ✓ Fish box slippage on the belt is negligible.

The electrical subsystem is used to drive the mechanical subsystem. The inverter with a DC voltage input controls the induction motor to generate sufficient torque to drive the mechanical subsystem, as shown in **Figure 2**.

Under the above assumptions, the *i*th mechanical driven system can be expressed by the following:

$$\tau_i = J_i\dot{\omega}_{i1} + f_i\omega_{i1} + \tau_{di}(t) \tag{1}$$

where $J_i = J_{i1} + J_{i2}$, $f_i = f_{i1} + f_{i2}$, $\tau_{di}(t)$ is a bounded external disturbance torque and τ_i is the sufficient torque to drive the mechanical subsystem of the *i*th conveyor, given as follows:

$$\tau_i = k_i u_i^* \tag{2}$$

where k_i is an amplifier gain and u_i^* is the DC voltage input of the *i*th inverter to create the desired torque τ_i. u_i^* is defined as a saturated control input of the *i*th conveyor as follows:

$$u_i^* = \begin{cases} u_{i\min} & \text{for} \quad u_i < u_{i\min} \\ u_0 & \text{for} \quad u_{i\min} \le u \le u_{i\max} \\ u_{i\max} & \text{for} \quad u_i > u_{i\max} \end{cases} \tag{3}$$

where u_i^* is the designed control input for the *i*th conveyor by the proposed controller, and u_{imin} and u_{imax} are the limited thresholds of the *i*th designed control inputs.

A dynamic FSS based on **Equation (1)** - **Equation (3)** can be expressed in the state space as follows:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\left(\mathbf{u}^* - \mathbf{d}(t)\right) \tag{4}$$

where $\mathbf{x} = \begin{bmatrix}\omega_{11} & \omega_{21} & \omega_{31}\end{bmatrix}^T$ is an angular velocity output vector of the FSS measured by encoders attached to the driving rollers, ω_{i1} is the angular velocity of the driving roller of the *i*th conveyor, $\mathbf{u}^* = sat(\mathbf{u}) = \begin{bmatrix}u_1^* & u_2^* & u_3^*\end{bmatrix}^T$ is a saturated control input vector, $\mathbf{d}(t) = \begin{bmatrix}d_1 & d_2 & d_3\end{bmatrix}^T$ is a bounded external disturbance vector with $d_i = \frac{\tau_{di}(t)}{k_i}$, and the unknown constant matrices $\mathbf{A}, \mathbf{B} \in \Re^{3\times3}$ are given as follows: $\mathbf{A} = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$,

$\mathbf{B}=\begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix}$ with $a_{ii}=-\frac{f_i}{J_i}$ and $b_{ii}=\frac{k_i}{J_i}$.

3. Controller Design

The control objective is to determine a designed control input vector $\mathbf{u}=\begin{bmatrix} u_1 & u_2 & u_3 \end{bmatrix}^T$ for a modified model reference adaptive system with saturated inputs and bounded disturbance, such that the angular velocity output vector tracks the output vector of a reference model.

A modified reference model used for its output vector to asymptotically track a trapezoidal-type reference input vector **r** is chosen as follows:

$$\dot{\mathbf{x}}_m=\mathbf{A}_m\mathbf{x}_m+\mathbf{B}_m\mathbf{r}+\dot{\mathbf{r}}+\lambda\mathbf{e} \tag{5}$$

$$\mathbf{e}=\mathbf{x}-\mathbf{x}_m \tag{6}$$

where $\lambda > 0$ is an error feedback gain, $\mathbf{x}_m=\begin{bmatrix} \omega_{m1} & \omega_{m2} & \omega_{m3} \end{bmatrix}^T$ is the angular velocity output vector of the modified reference model, ω_{mi} is the ith reference angular velocity, **e** is a modeling error vector, $\mathbf{r}=\begin{bmatrix} r_1 & r_2 & r_3 \end{bmatrix}^T$ is the reference angular velocity input vector, and $\mathbf{A}_m,\mathbf{B}_m\in\Re^{3\times3}$ are given as follows:

$$\mathbf{A}_m=\begin{bmatrix} a_{m1} & 0 & 0 \\ 0 & a_{m2} & 0 \\ 0 & 0 & a_{m3} \end{bmatrix},\ \mathbf{B}_m=\begin{bmatrix} b_{m1} & 0 & 0 \\ 0 & b_{m2} & 0 \\ 0 & 0 & b_{m3} \end{bmatrix}$$

where a_{mi}, b_{mi} are reference model parameters chosen to satisfy **Assumptions 2** and **3** as follows.

- **Assumption 2:** Given a known Hurwitz matrix $\mathbf{A}_m\in\Re^{3\times3}$ and a known matrix $\mathbf{B}_m\in\Re^{3\times3}$ of full rank, there exists an unknown control gain matrix $\mathbf{K}\in\Re^{3\times3}$ and an unknown positive definite diagonal constant matrix $\Lambda\in\Re^{3\times3}$ such that the following equations hold:

$$\begin{cases} \mathbf{A}=\mathbf{A}_m-\mathbf{BK} \\ \mathbf{B}=\mathbf{B}_m\Lambda \end{cases} \tag{7}$$

- **Assumption 3:** A positive symmetric definite matrix $\mathbf{P}=\mathbf{P}^T>0$ is the solution of the following Lyapunov equation:

$$\mathbf{A}_m^T\mathbf{P}+\mathbf{PA}_m=-\mathbf{Q}_m \tag{8}$$

where $\mathbf{Q}_m$ is a positive definite matrix.

Substituting **Equation (7)** into **Equation (4)**, adding and subtracting $\mathbf{B}_m\mathbf{r}$, $\dot{\mathbf{r}}$ yields

$$\dot{\mathbf{x}}=\mathbf{A}_m\mathbf{x}+\mathbf{B}_m\mathbf{r}+\dot{\mathbf{r}}+\mathbf{B}_m\Lambda\left(\mathbf{u}^*-\mathbf{u}_r\right) \tag{9}$$

where $\Phi=\Lambda^{-1}$, $\Omega=\left(\mathbf{B}_m\Lambda\right)^{-1}$, and $\mathbf{u}_r$ is an ideal control input vector as follows:

$$\mathbf{u}_r=\mathbf{Kx}+\Phi\mathbf{r}+\Omega\dot{\mathbf{r}}+\mathbf{d}(t) \tag{10}$$

The first-order time derivative of e is given by

$$\dot{\mathbf{e}}=\left(\mathbf{A}_m-\lambda\mathbf{I}\right)\mathbf{e}+\mathbf{B}_m\Lambda\left(\mathbf{u}^*-\mathbf{u}_r\right) \tag{11}$$

If $\mathbf{u}^*=\mathbf{u}_r$, $\dot{\mathbf{e}}=\left(\mathbf{A}_m-\lambda\mathbf{I}\right)\mathbf{e}$. Because $\mathbf{A}_m$ and $\left(\mathbf{A}_m-\lambda\mathbf{I}\right)$ are Hurwitz matrices, it can be concluded that $\mathbf{e}\to 0$ as $t\to\infty$. This implies that the plant in **Equation (4)** can asymptotically track the reference model in **Equation (5)**. However, the ideal control input vector cannot be implemented, because the matrices **K**, **Φ**, and **Ω,** and the disturbance vector **d**(t) are unknown. Therefore, a designed control input vector **u** is chosen as an estimate of $\mathbf{u}_r$ in the following form:

$$\mathbf{u}=\hat{\mathbf{K}}\mathbf{x}+\hat{\Phi}\mathbf{r}+\hat{\Omega}\dot{\mathbf{r}}+\hat{\mathbf{d}}(t) \tag{12}$$

where $\hat{\mathbf{K}},\hat{\Phi},\hat{\Omega}$ are estimations of unknown control gain matrices $\mathbf{K},\Phi,\Omega$ and $\hat{\mathbf{d}}\in\Re^3$ is an estimated vector of an unknown constant vector $\bar{\mathbf{d}}$, which is the averaged-value vector of **d**(t) in **Equation (4)**.

A saturated input error vector is defined as

$$\Delta\mathbf{u}=\mathbf{u}-\mathbf{u}^* \tag{13}$$

From **Equation (10)** - **Equation (13)**, the first-order time derivative of **e** is given as

$$\begin{aligned}\dot{\mathbf{e}}=&\left(\mathbf{A}_m-\lambda\mathbf{I}\right)\mathbf{e}+\mathbf{B}_m\Lambda\left(\tilde{\mathbf{K}}\mathbf{x}+\tilde{\Phi}\mathbf{r}+\tilde{\Omega}\dot{\mathbf{r}}+\tilde{\mathbf{d}}\right)\\&+\mathbf{B}_m\Lambda\left(\bar{\mathbf{d}}-\mathbf{d}-\Delta\mathbf{u}\right)\end{aligned} \tag{14}$$

where $\tilde{\mathbf{K}}=\hat{\mathbf{K}}-\mathbf{K}$, $\tilde{\Phi}=\hat{\Phi}-\Phi$, $\tilde{\Omega}=\hat{\Omega}-\Omega$, and $\tilde{\mathbf{d}}=\hat{\mathbf{d}}-\bar{\mathbf{d}}$.

To remove the effect of the saturated input, an auxiliary error vector $\mathbf{e}_\Delta$ is defined as

$$\dot{\mathbf{e}}_\Delta=\left(\mathbf{A}_m-\lambda\mathbf{I}\right)\mathbf{e}_\Delta-\hat{\mathbf{K}}_\Delta\Delta\mathbf{u} \tag{15}$$

where $(\mathbf{A}_m-\lambda\mathbf{I})$ is a stable Hurwitz matrix and $\hat{\mathbf{K}}_\Delta\in\Re^{3\times3}$ is the adaptable parameter matrix.

Therefore, a new error vector is defined as follows:

$$\mathbf{e}_u=\mathbf{e}-\mathbf{e}_\Delta \tag{16}$$

From **Equation (14) ~ Equation (16)**, the first-order time derivative of $\mathbf{e}_u$ is given as

$$\begin{aligned}\dot{\mathbf{e}}_u=&\left(\mathbf{A}_m-\lambda\mathbf{I}\right)\mathbf{e}_u+\mathbf{B}_m\Lambda\left(\tilde{\mathbf{K}}\mathbf{x}+\tilde{\Phi}\mathbf{r}+\tilde{\Omega}\dot{\mathbf{r}}+\tilde{\mathbf{d}}\right)\\&+\tilde{\mathbf{K}}_\Delta\Delta\mathbf{u}+\mathbf{B}_m\Lambda\left(\bar{\mathbf{d}}-\mathbf{d}\right)\end{aligned} \tag{17}$$

where $\tilde{\mathbf{K}}_\Delta = \hat{\mathbf{K}}_\Delta - \mathbf{B}_m\mathbf{\Lambda}$.

The control gains $\hat{\mathbf{K}},\hat{\mathbf{\Phi}},\hat{\mathbf{\Omega}},\hat{\mathbf{d}}$ in **Equation (12)** are estimated by update laws based on a σ-modification that will be designed in **Theorem 1.**

Theorem 1: An M-MRAC system of **Equation (4)** is stable as long as a designed control input vector of **Equation (12)** is given and update laws using a σ-modification are given as

$$\begin{cases} \dot{\hat{\mathbf{K}}} = -\sigma\hat{\mathbf{K}} - \gamma_1\mathbf{B}_m^T\mathbf{P}\mathbf{e}_u\mathbf{x}^T \\ \dot{\hat{\mathbf{\Phi}}} = -\sigma\hat{\mathbf{\Phi}} - \gamma_1\mathbf{B}_m^T\mathbf{P}\mathbf{e}_u\mathbf{r}^T \\ \dot{\hat{\mathbf{\Omega}}} = -\sigma\hat{\mathbf{\Omega}} - \gamma_1\mathbf{B}_m^T\mathbf{P}\mathbf{e}_u\dot{\mathbf{r}}^T \end{cases} \tag{18}$$

$$\dot{\hat{\mathbf{K}}}_\Delta = -\sigma\hat{\mathbf{K}}_\Delta - \gamma_2\mathbf{P}\mathbf{e}_u\Delta\mathbf{u}^T \text{ , and} \tag{19}$$

$$\dot{\hat{\mathbf{d}}} = -\sigma\hat{\mathbf{d}} - \gamma_3\mathbf{B}_m^T\mathbf{P}\mathbf{e}_u \tag{20}$$

where $\gamma_1,\gamma_2,\gamma_3 > 0$ are the adaptation rates, and σ is a design parameter.

[**Proof of Theorem 1**]**:** A candidate Lyapunov function is chosen to analyze the stability of the system as follows:

$$V(t) = \mathbf{e}_u^T\mathbf{P}\mathbf{e}_u + \frac{1}{\gamma_1}trace\left(\tilde{\mathbf{K}}^T\mathbf{\Lambda}\tilde{\mathbf{K}} + \tilde{\mathbf{\Phi}}^T\mathbf{\Lambda}\tilde{\mathbf{\Phi}} + \tilde{\mathbf{\Omega}}^T\mathbf{\Lambda}\tilde{\mathbf{\Omega}}\right) + \frac{1}{\gamma_2}trace\left(\tilde{\mathbf{K}}_\Delta^T\tilde{\mathbf{K}}_\Delta\right) + \frac{1}{\gamma_3}trace\left(\tilde{\mathbf{d}}^T\tilde{\mathbf{d}}\mathbf{\Lambda}\right) \geq 0 \tag{21}$$

The first-order time derivative of V(t) is given as

$$\begin{aligned} \dot{V} = &-\mathbf{e}_u^T\left(\mathbf{Q}_m + 2\lambda\mathbf{P}\right)\mathbf{e}_u + 2trace\left(\tilde{\mathbf{K}}^T\mathbf{\Lambda}\mathbf{B}_m^T\mathbf{P}\mathbf{e}_u\mathbf{x}^T + \tilde{\mathbf{\Phi}}^T\mathbf{\Lambda}\mathbf{B}_m^T\mathbf{P}\mathbf{e}_u\mathbf{r}^T + \tilde{\mathbf{\Omega}}^T\mathbf{\Lambda}\mathbf{B}_m^T\mathbf{P}\mathbf{e}_u\dot{\mathbf{r}}^T + \tilde{\mathbf{K}}_\Delta^T\mathbf{P}\mathbf{e}_u\Delta\mathbf{u}^T\right) \\ &+ 2\tilde{\mathbf{d}}^T\mathbf{\Lambda}\mathbf{B}_m^T\mathbf{P}\mathbf{e}_u + \frac{2}{\gamma_1}trace\left(\tilde{\mathbf{K}}^T\mathbf{\Lambda}\dot{\hat{\mathbf{K}}} + \tilde{\mathbf{\Phi}}^T\mathbf{\Lambda}\dot{\hat{\mathbf{\Phi}}} + \tilde{\mathbf{\Omega}}^T\mathbf{\Lambda}\dot{\hat{\mathbf{\Omega}}}\right) \\ &+ \frac{2}{\gamma_2}trace\left(\tilde{\mathbf{K}}_\Delta^T\dot{\hat{\mathbf{K}}}_\Delta\right) + \frac{2}{\gamma_3}trace\left(\tilde{\mathbf{d}}^T\dot{\hat{\mathbf{d}}}\mathbf{\Lambda}\right) + 2\mathbf{e}_u^T\mathbf{P}\mathbf{B}_m\mathbf{\Lambda}\left(\bar{\mathbf{d}} - \mathbf{d}\right) \end{aligned} \tag{22}$$

Using **Equation (18)** - **Equation (20)** yields

$$\begin{aligned} \dot{V} = &-\mathbf{e}_u^T\left(\mathbf{Q}_m + 2\lambda\mathbf{P}\right)\mathbf{e}_u - \frac{2\sigma}{\gamma_1}trace\left(\tilde{\mathbf{K}}^T\mathbf{\Lambda}\hat{\mathbf{K}} + \tilde{\mathbf{\Phi}}^T\mathbf{\Lambda}\hat{\mathbf{\Phi}} + \tilde{\mathbf{\Omega}}^T\mathbf{\Lambda}\hat{\mathbf{\Omega}}\right) \\ &- \frac{2\sigma}{\gamma_2}trace\left(\tilde{\mathbf{K}}_\Delta^T\hat{\mathbf{K}}_\Delta\right) - \frac{2\sigma}{\gamma_3}trace\left(\tilde{\mathbf{d}}^T\hat{\mathbf{d}}\mathbf{\Lambda}\right) + 2\mathbf{e}_u^T\mathbf{P}\mathbf{B}_m\mathbf{\Lambda}\left(\bar{\mathbf{d}} - \mathbf{d}\right) \end{aligned} \tag{23}$$

Using the Rayleigh principle, **Equation (23)** can be written as follows:

$$\begin{aligned} \dot{V} \leq &-a_1\left\|\mathbf{e}_u\right\|^2 + 2\left\|\mathbf{e}_u\right\|d_* - \frac{2\sigma}{\gamma_1}\lambda_{\min}\left(\mathbf{\Lambda}\right)\left(\left\|\tilde{\mathbf{K}}\right\|_F^2 + \left\|\tilde{\mathbf{\Phi}}\right\|_F^2 + \left\|\tilde{\mathbf{\Omega}}\right\|_F^2\right) \\ &- \frac{2\sigma}{\gamma_2}\lambda_{\min}\left(\mathbf{\Lambda}\right)\left\|\tilde{\mathbf{K}}_\Delta\right\|_F^2 - \frac{2\sigma}{\gamma_3}\lambda_{\min}\left(\mathbf{\Lambda}\right)\left\|\tilde{\mathbf{d}}\right\|^2 \\ \leq &-a_1\left\|\mathbf{e}_u\right\|^2 + 2\left\|\mathbf{e}_u\right\|d_* + c \end{aligned} \tag{24}$$

where $a_1 = \lambda_{\min}\left(\mathbf{Q}_m\right) + 2\lambda_{\min}\left(\mathbf{P}\right)\lambda > 0,\ d_* = \left\|\mathbf{P}\mathbf{B}_m\Lambda\left(\mathbf{d} - \bar{\mathbf{d}}\right)\right\| \in l_\infty$, $\|\cdot\|_F$ is the Frobenius norm and c is given as follows:

$$c = -\frac{2\sigma}{\gamma_1}\lambda_{\min}\left(\mathbf{\Lambda}\right)\left(\left\|\tilde{\mathbf{K}}\right\|_F^2 + \left\|\tilde{\mathbf{\Phi}}\right\|_F^2 + \left\|\tilde{\mathbf{\Omega}}\right\|_F^2\right) - \frac{2\sigma}{\gamma_2}\lambda_{\min}\left(\mathbf{\Lambda}\right)\left\|\tilde{\mathbf{K}}_\Delta\right\|_F^2 - \frac{2\sigma}{\gamma_3}\lambda_{\min}\left(\mathbf{\Lambda}\right)\left\|\tilde{\mathbf{d}}\right\|^2 \tag{25}$$

$\dot{V}(t)$ in **Equation (24)** is negative semi-definite outside the compact set

$$H = \left\{\left(\mathbf{e}_u,\tilde{\mathbf{K}},\tilde{\mathbf{\Phi}},\tilde{\mathbf{\Omega}},\tilde{\mathbf{K}}_\Delta,\tilde{\mathbf{d}}\right) : a_1\left\|\mathbf{e}_u\right\|^2 - 2\left\|\mathbf{e}_u\right\|d^* > c\right\} \tag{26}$$

This implies that $\mathbf{e}_u,\hat{\mathbf{K}},\hat{\mathbf{\Phi}},\hat{\mathbf{\Omega}},\hat{\mathbf{K}}_\Delta$, and $\hat{\mathbf{d}}$ are bounded from **Equations (21)**, **(24)**, and **(26)**, and $\mathbf{e}_u \to 0$ as $t \to \infty$ by Barbalat's lemma. Hence, $\mathbf{e} \to \mathbf{e}_\Delta$ and **e** is bounded if and only if $\mathbf{e}_\Delta$ is also bounded. The boundedness $\mathbf{e}_\Delta$ is proven as follows:

A candidate of Lyapunov function is chosen as

$$W = \mathbf{e}_\Delta^T\mathbf{P}\mathbf{e}_\Delta \geq 0 \tag{27}$$

Using **Equations (15)** and **(27)**, the first-order time derivative of W is given as

$$\begin{aligned} \dot{W} &= -\mathbf{e}_\Delta^T\left(\mathbf{Q}_m + 2\lambda\mathbf{P}\right)\mathbf{e}_\Delta - 2\mathbf{e}_\Delta^T\mathbf{P}\hat{\mathbf{K}}_\Delta\Delta\mathbf{u} \\ &\leq -\left\|\mathbf{e}_\Delta\right\|^2 a_1 + \lambda_3 \leq \lambda_2 W + \lambda_3 \end{aligned} \tag{28}$$

where $\lambda_3 = 2\left\|-\mathbf{e}_\Delta^T\mathbf{P}\hat{\mathbf{K}}_\Delta^T\Delta\mathbf{u}\right\| \geq 0$ and $\lambda_2 = \dfrac{a_1}{\lambda_{\max}\left(\mathbf{P}\right)} > 0$.

By using the Gronwall–Bellman Inequality, **Equation (28)** implies that

$$W \leq \left[W\left(\mathbf{e}_\Delta(0)\right) - \frac{\lambda_3}{\lambda_2}\right]\exp\left(-\lambda_2 t\right) + \frac{\lambda_3}{\lambda_2} \tag{29}$$

Using **Equations (27)** and **(29)**, the following are obtained:

$$\lim_{t\to\infty}\mathbf{e}_\Delta^T\mathbf{P}\mathbf{e}_\Delta \leq \frac{\lambda_3}{\lambda_2} \tag{30}$$

$$\frac{\lambda_3}{\lambda_2} \geq \lim_{t\to\infty}\mathbf{e}_\Delta^T\mathbf{P}\mathbf{e}_\Delta \geq \lambda_{\min}\left(\mathbf{P}\right)\lim_{t\to\infty}\left\|\mathbf{e}_\Delta\right\|^2 \text{ , and} \tag{31}$$

$$\lim_{t\to\infty}\left\|\mathbf{e}_\Delta\right\| \leq \sqrt{\frac{\lambda_3}{\lambda_2\lambda_{\min}\left(\mathbf{P}\right)}} \tag{32}$$

It can be proven that $\mathbf{e}_\Delta$ is also bounded.

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The block diagram of the proposed controller is shown in **Figure 3**.

A comparison of M-MRAC with CMRAC is given in **Table 1.**

Table 1: Comparison of M-MRAC with CMRAC

	CMRAC	M–MRAC
Ref. model	$\dot{\mathbf{x}}_m = \mathbf{A}_m\mathbf{x}_m + \mathbf{B}_m\mathbf{r} + \dot{\mathbf{r}}$	$\dot{\mathbf{x}}_m = \mathbf{A}_m\mathbf{x}_m + \mathbf{B}_m\mathbf{r} + \dot{\mathbf{r}} + \lambda\mathbf{e}$
Update laws	$\begin{cases}\dot{\hat{\mathbf{K}}} = -\gamma\mathbf{e}^T\mathbf{P}\mathbf{B}_m\mathbf{x},\\ \dot{\hat{\mathbf{\Phi}}} = -\gamma\mathbf{e}^T\mathbf{P}\mathbf{B}_m\mathbf{r},\\ \dot{\hat{\mathbf{\Omega}}} = -\gamma\mathbf{e}^T\mathbf{P}\mathbf{B}_m\dot{\mathbf{r}},\\ \dot{\hat{\mathbf{d}}} = -\gamma\mathbf{e}^T\mathbf{P}\mathbf{B}_m.\end{cases}$	$\begin{cases}\dot{\hat{\mathbf{K}}} = -\sigma\hat{\mathbf{K}} - \gamma\mathbf{e}^T\mathbf{P}\mathbf{B}_m\mathbf{x},\\ \dot{\hat{\mathbf{\Phi}}} = -\sigma\hat{\mathbf{\Phi}} - \gamma\mathbf{e}^T\mathbf{P}\mathbf{B}_m\mathbf{r},\\ \dot{\hat{\mathbf{\Omega}}} = -\sigma\hat{\mathbf{\Omega}} - \gamma\mathbf{e}^T\mathbf{P}\mathbf{B}_m\dot{\mathbf{r}},\\ \dot{\hat{\mathbf{d}}} = -\sigma\hat{\mathbf{d}} - \gamma\mathbf{e}^T\mathbf{P}\mathbf{B}_m.\end{cases}$

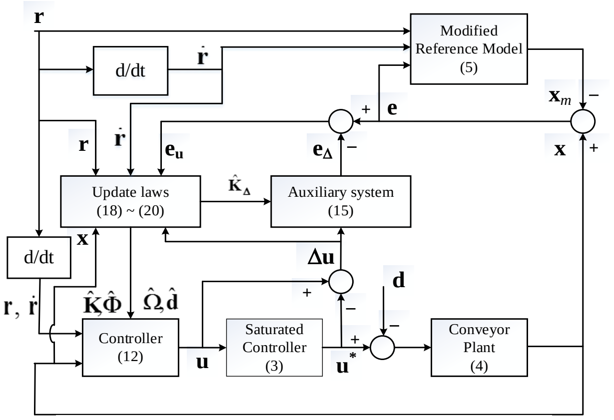

Figure 3: Block diagram of the proposed controller

4. Experimental Results

To evaluate the effectiveness of the proposed controller (M-MRAC) with σ-modification and compare it with the CMRAC, an experiment was carried out under the following conditions.

The initial values of the state variables and the controller inputs were set to zero. The input voltages of the inverters considered as control inputs of the proposed controller can vary in range from $u_{1min} = u_{2min} = u_{3min} = 0$ V to $u_{1max} = u_{2max} = u_{3max} = 5$ V.

The parameters of the modified model reference system was given by $a_{m1} = a_{m2} = a_{m3} = -30$ and $b_{m1} = b_{m2} = b_{m3} = 30$. The error feedback gain was chosen as $\lambda = 10$, the fixed controller gains were chosen as $\gamma_2 = 3.3$, $\gamma_3 = 1.3$, $\sigma = 0.03$, and the positive symmetric definite matrix was chosen as $\mathbf{P} = diag\left(\begin{bmatrix}10^{-8} & 10^{-8} & 10^{-8}\end{bmatrix}\right)$. The reference inputs for the conveyor plant were the angular velocity inputs and are given in **Figure 4**.

To demonstrate the effectiveness of the proposed controller, the following three cases were considered.

Case 1: The adaptation rates of both the CMRAC and M-MRAC are set to $\gamma_1 = 1.67$. It can be seen that both the output of the proposed M-MRAC $x_1(t)$ and the output of the CMRAC $x_{1M}(t)$ for the 1st conveyor track the reference input $r_1(t)$, as shown in **Figure 5**. However, the output amplitude of the CMRAC varies more strongly than that of the proposed M-MRAC. High-frequency oscillations are generated in the control input signal $u_{1M}(t)$ of the CMRAC, whereas the control input signal $u_1(t)$ of the proposed M-MRAC remains nearly unchanged, as shown in **Figure 6**, when the angular velocity output reaches 42.1 rad/s in **Figure 5**.

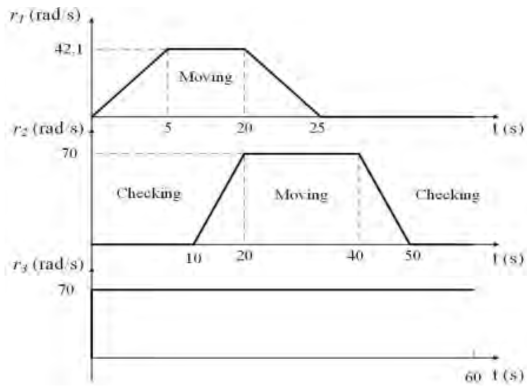

Figure 4: Velocity profiles of all conveyors

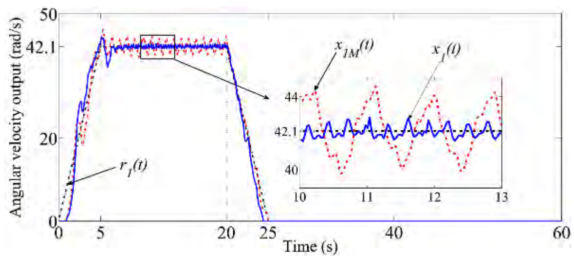

Figure 5: Output of the CMRAC and M-MRAC for the 1st conveyor

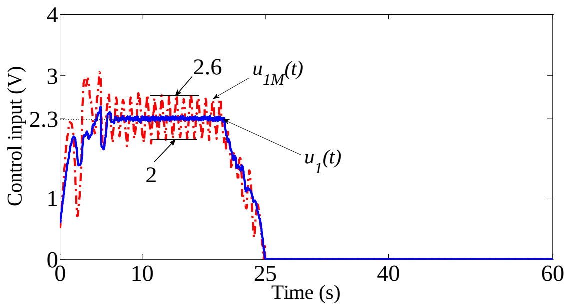

Figure 6: Control input of the CMRAC and M-MRAC for the 1st conveyor

Case 2: The adaptation rates of both the CMRAC and M-MRAC are set to $\gamma_1 = 10$. The angular velocity output $x_2(t)$ of the proposed M-MRAC for the 2nd conveyor tracks the reference input $r_2(t)$ with a small error (from −1.1 rad/s to +1.3 rad/s), whereas the angular velocity output $x_{2M}(t)$ of the CMRAC for the 2nd conveyor tracks the reference input $r_2(t)$ with an error from −2.3 rad/s to +2.5 rad/s in **Figure 7**. The control input $u_{2M}(t)$ of the CMRAC also oscillates with higher frequency and amplitude than the control input $u_2(t)$ of the proposed M-MRAC, as shown in **Figure 8**. The maximum and minimum values of the control input of the CMRAC are 4.1 V and 3.72 V, respectively, and the average value of the control input is 3.88 V. The control input for the proposed M-MRAC varies more slowly than that for the CMRAC. Therefore, the performance of the proposed M-MRAC is better than that of the CMRAC in this case.

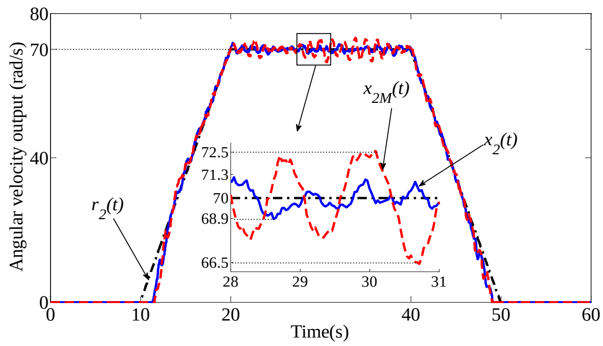

Figure 7: Output of the CMRAC and the M-MRAC for the 2nd conveyor

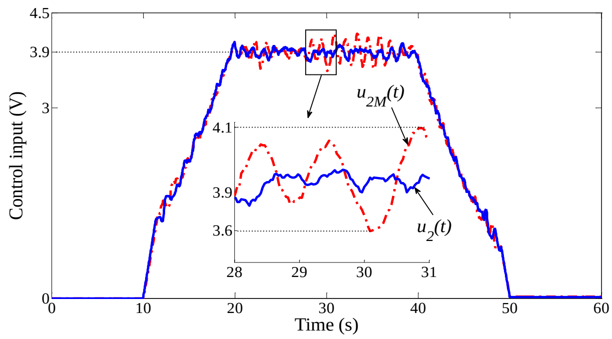

Figure 8: Control input of the CMRAC and M-MRAC for the 2nd conveyor

Case 3: Similarly, the adaptation rates of both the CMRAC and M-MRAC are set to γ_1 = 6.67. The angular velocity output $x_3(t)$ of the proposed M-MRAC for the 3rd conveyor also tracks the reference input $r_3(t)$ better than the angular velocity output $x_{3M}(t)$ of the CMRAC, as shown in **Figure 9**. Because the reference input $r_3(t)$ is a step type, the control input signal amplitudes of both the proposed M-MRAC and the CMRAC are large (15.5 V and 12.5 V, respectively), as shown in **Figure 10**. Therefore, the saturated control input $u_3^*(t)$ is set to u_{3max} = 5 V, and the angular velocity outputs of both the proposed M-MRAC and the CMRAC reach 90.3 rad/s in **Figure 9**.

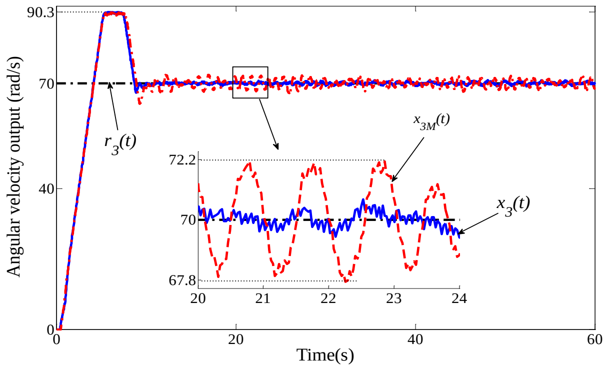

Figure 9: Output of the CMRAC and the M–MRAC for the 3rd conveyor

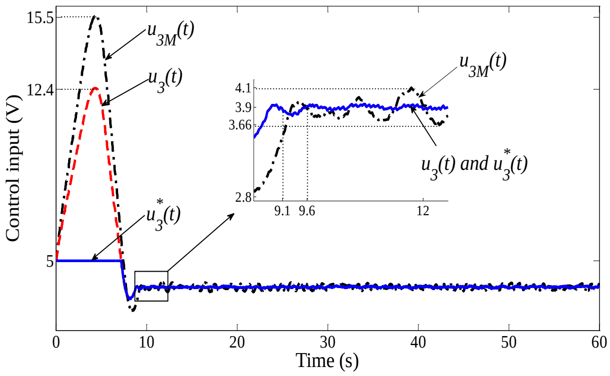

Figure 10: Control input of the CMRAC and the M–MRAC for the 3rd conveyor

The tracking performance of the proposed M-MRAC controller versus the CMRAC is given in **Table 2**.

Table 2: Modeling error of the CMRAC and M–MRAC

Parameters		CMRAC	M–MRAC
Case 1	Output (rad/s)	$42.1\pm^{1.9}_{2.1}$	$42.1\pm^{0.9}_{0.5}$
	Control input (V)	2.3V ± 0.3	$2.3V\pm^{0.04}_{0.07}$
	Adaptation rate	1.67	1.67
	Modeling error	5%	2.1%
Case 2	Output (rad/s)	$70\pm^{2.5}_{3.5}$	$70\pm^{1.3}_{1.1}$
	Control input (V)	$3.9V\pm^{0.2}_{0.3}$	$3.9V\pm^{0.1}_{0.16}$
	Adaptation rate	10	10
	Modeling error	5%	1.86%
Case 3	Output (rad/s)	70 ± 2	$70\pm^{0.6}_{0.8}$
	Control input (V)	$3.9V\pm^{0.12}_{0.24}$	$3.9V\pm^{0.04}_{0.07}$
	Adaptation rate	6.67	6.67
	Modeling error	2.9%	1.14%

5. Conclusion

A modified-model reference adaptive controller for belt conveyors in an FSS with uncertainty parameters, input saturation, and bounded disturbances was proposed. The feedback of the modeling error signal in the proposed M-MRAC controller obtained a smaller modeling error than that in the CMRAC. The tracking performance of the proposed M−MRAC showed a better improvement in both the transient and asymptotic states than that of the CMRAC, and the high-frequency elements in the control input signals were reduced in the proposed M−MRAC when the adaptation rate was increased. The error dynamics under the input saturation were compensated by the auxiliary output error. The experimental results showed that the proposed M-MRAC became more effective than the CMRAC when the adaptation rate was large and the error feedback gain was suitably selected.

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