Analytical solution of the Cattaneo‑Vernotte equation (non-Fourier heat conduction)

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Abstract: The theory of Fourier heat conduction predicts accurately the temperature profiles of a system in a non-equilibrium steady state. However, in the case of transient states at the nanoscale, its applicability is significantly limited. The limitation of the classical Fourier's theory was overcome by C. Cattaneo and P. Vernotte who developed the theory of non-Fourier heat conduction in 1958. Although this new theory has been used in various thermal science areas, it requires considerable mathematical skills for calculating analytical solutions. The aim of this study was the identification of a newer and a simpler type of solution for the hyperbolic partial differential equations of the non-Fourier heat conduction. This constitutes the first trial in a series of planned studies. By inspecting each term included in the proposed solution, the theoretical feasibility of the solution was achieved. The new analytical solution for the non-Fourier heat conduction is a simple exponential function that is compared to the existing data for justification. Although the proposed solution partially satisfies the Cattaneo‑Vernotte equation, it cannot simulate a thermal wave behavior. However, the results of this study indicate that it is possible to obtain the theoretical solution of the Cattaneo‑Vernotte equation by improving the form of the proposed solution.

Keywords: Cattaneo‑Vernotte equation, Hyperbolic type partial differential equation, Non-Fourier heat conduction

1. Introduction

A heat wave, also known as a thermal wave, constitutes a subject of intense research but it is still an ambiguous concept in thermal science and engineering. At present, the phenomena elicited by a thermal wave are being applied to nondestructive examinations (NDE) [1] or skin care applications [2]. These applications utilize the physical characteristics of materials given that the thermal conductance and the thermal relaxation times are different in a variety of materials. Thermal conductance is the rate of heat absorption and thermal relaxation is the rate of heat dissipation.

The concept of non-Fourier heat conduction resulted from the newly issued heat conduction mechanism that modified Fourier's heat conduction theory, which was issued by C. Cattaneo and P. Vernotte independently in 1958 [3]. The defect of the Fourier heat conduction equation was originally indicated by J. C. Maxwell in 1867 with the comment that it implies the use of an infinite propagation speed for a thermal disturbance in a medium [4].

The heat conduction phenomena in solids are well predicted by the classical Fourier heat conduction equation in almost all engineering cases. Nevertheless, it cannot be applied to specific scientific cases that deviate from the conventional ones, which usually include the heat transfer phenomena at the nanoscale, periodic heat fluxes, laser heating, or the presence of a cryogenic region [5]. According to recent studies, the wave characteristics appear in a heat transfer process under some specialized conditions or applications.

The term "wave" implies that some physical quantities possess specific characteristics, such as the wavefront, and its reflection or refraction at a boundary. Additionally, if a heat transfer phenomenon can be treated by the wave theory, matter should possess the dual properties of wave and particle simultaneously as heat transfer in solids can be interpreted by an imaginary particle known as the phonon that possesses wave-like properties [6].

On the other hand, some studies argue that the concept of a thermal wave was not valid because there are no observations of the wavelike characteristics, and because an applied thermal transient does not travel fast in a medium as a wave does, but rather diffuses in it [7]. However, this study

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is not concerned on whether a thermal disturbance is wave-like (or not) since the purpose herein is to find the simple type of the mathematical solution for the governing equation of a thermal wave.

Based on the investigations by the authors in prior studies on thermal waves [8][9], a thermal disturbance in any material is expected to be propagated with an infinite velocity. In the real world, the infinite propagation speed of a thermal wave is not possible although it can be assumed to be reasonable under specific, constrained conditions.

According to the study of Marvin Chest [10], the square of the thermal wave velocity (C_T) is one third of the square of the phonon velocity (C_A), that is $C_T^2 = C_A^2/3$. The thermal wave, which is the main feature of non-Fourier heat conduction, became after this initial study the focus of research work relevant to thermal phenomena.

Specifically, in reference to typical studies on the non-Fourier heat conduction, J. Gembarovic et al. [11] presented an analytical solution for heat pulse propagation in a one- dimensional system using the Dirac-delta function, the Heaviside unit step function, the Laplace transform, and the Inverse transform. D. D. Joseph et al. [3] derived the governing equations and introduced the procedures to calculate the temperature field and the heat flux in a system. However, he did indicate that Fourier's heat conduction equation has still practical authority in science and engineering because the thermal relaxation time is extremely short compared to the time scale of events in our daily lives, which is the distinctive difference between the Fourier and non-Fourier heat conductions.

K. K. Tamma et al. [4] published a study on the thermal transport phenomena at macroscale and microscale systems. More recently, significant additional attention has been paid on non-Fourier heat conduction due to its potential applications to the fields of laser heating processes [12] and microscale heat transfer [13]. C. W. Chang et al. indicated that the prediction of heat conduction based on Fourier's heat conduction theory largely deviated from the experimental data when a system had a low dimension (for example, one dimension, such as the case of a carbon nanotube or a chain) [14].

Megan Jaunich et al. performed an experiment to investigate the temperature distribution when a laser beam irradiated the skin tissue [15]. According to their findings, the experimental results disagreed with the predictions of Fourier's heat conduction equation but agreed with the calculated results based on the non-Fourier's heat conduction equation.

As mentioned previously, despite the fact that so many thermal scientists have concentrated their efforts on non-Fourier heat conduction phenomena, it is somewhat difficult to com-

prehend the theory thoroughly. To understand it, elaborate mathematical skills are required.

Accordance to the authors' viewpoints, the previous analytical studies require a profound mathematical knowledge to understand the non-Fourier heat conduction phenomena, an issue that still presents difficulties to engineers. This matter constitutes the real motivation of the study in identifying a simple analytical solution for the partial differential equation of the non-Fourier's heat conduction.

The starting point of this study is the assumption of an appropriate solution for the partial differential equation of the non-Fourier's heat conduction that is as simple as possible, which is the only method to find a solution for a given differential equation. If this assumed solution satisfies the given partial differential equation, there is no reason that it cannot be accepted as the final solution. In this study, the temporally and spatially varying exponential function was assumed as a solution, and it was shown that it satisfies the differential equation of the non-Fourier's heat conduction.

The physical conformity of an obtained analytical solution was not considered in this study because this attempt constitutes the first trial to identify a simplified form of the theoretical solution that meets the mathematical requirements. The exact solution that satisfies the mathematical and physical behaviors is then formulated by combining any discontinuity function (e.g., step, Dirac-delta, etc.) and the obtained solution.

2. Non-Fourier heat conduction

As mentioned in the previous section, the partial differential equation for the non-Fourier heat conduction was presented by C. Cattaneo and P. Vernotte and it hereafter referred to as the Cattaneo‑Vernotte (C‑V) equation. The C‑V equation resulted from the modification of Fourier's heat conduction equation [**Equation (1)**] with consideration of the time interval between a heat flux and a temperature gradient (1).

$$\frac{\partial^2 T(x,t)}{\partial t^2} = \alpha \frac{\partial^2 T(x,t)}{\partial x^2} \tag{1}$$

The C‑V equation is based on the finite propagation speed of a thermal disturbance at the boundary surface of a system, which means that the present heat flux should be determined with the use of the temperature gradient at any earlier time, τ. Therefore, the C‑V equation is more complicated compared to **Equation (1)**.

$$\frac{\partial^2 T(x,t)}{\partial t^2} + \frac{1}{\tau}\frac{\partial T(x,t)}{\partial t} = \frac{\alpha}{\tau}\frac{\partial^2 T(x,t)}{\partial x^2} \tag{2}$$

$$\frac{\partial^2 q(x,t)}{\partial t^2} + \frac{1}{\tau}\frac{\partial q(x,t)}{\partial t} = \frac{\alpha}{\tau}\frac{\partial^2 q(x,t)}{\partial x^2} \qquad (3)$$

In the adopted procedure that leads to **Equation (2)** and **Equation (3)**, only a one-dimensional system was considered for simplicity, as the purpose of this study was to evaluate the feasibility of the proposed solution.

In **Equation (1)**-**Equation (3)**, α is the thermal diffusivity with units of m^2/s, and τ is the thermal relaxation time, which indicates the time required to communicate among neighboring constituents on any thermal disturbance that occurred in their surroundings. As α/τ has units of m^2/s^2, that is, the square root of α/τ has units of (m/s), it can be regarded as the propagation speed of a thermal wave mentioned in the previous section, and it will be denoted as C_T. Ignoring the existence of the thermal relaxation time τ in the induction process leads to the conversion of **Equation (2)** back in the form of **Equation (1)**. This means that the C-V equation is more general than the conventional Fourier's heat conduction equation.

Equation (2) and **Equation (3)** are known as the hyperbolic partial differential equations, and the procedure used for their derivation was described by Jin *et al.* and D. Y. Tzou **[16][17]**. **Equation (2)** is the partial differential equation denoting the temperature profile in a system, and **Equation (3)** denotes the corresponding profile for the heat flux. Since the temperature profile in a system is usually a matter of interest when a thermal analysis is performed, this study is pursued using **Equation (2)**.

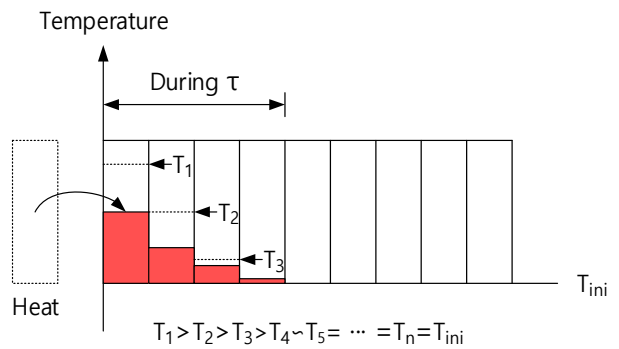## 2.1 Mechanism of a transient heat transfer

For finding the analytical solution of **Equation (2)**, the heat transfer mechanism as shown in **Figure 1** was devised. The initial state is shown in **Figure 1 (a)**; the whole system is maintained at any initial temperature ($t<0$). As shown in **Figure 1 (b)**, the left surface temperature is suddenly in- creased to T_{hot} at $t=0$, then the heat flow will be caused by the temperature gradient between both surfaces of the first shell.

The heat energy flowed into the 1st shell will increase its own temperature and creates the temperature difference between itself and the 2nd shell. The created temperature gradient between the 1st and the 2nd shells causes another heat flow. Certainly, these two procedures, which are the temperature rise of itself and the heat transfer to the neighboring shell under an initial temperature, will be occurred at the same time.

The 1st shell consumes a part of the inflow heat for its own temperature raise and transfers the rest of it to the 2nd shell. Due to the heat transferred from the 1st shell, the 2nd shell will also raise its own temperature and transfer a part of it to the 3rd shell and so on.
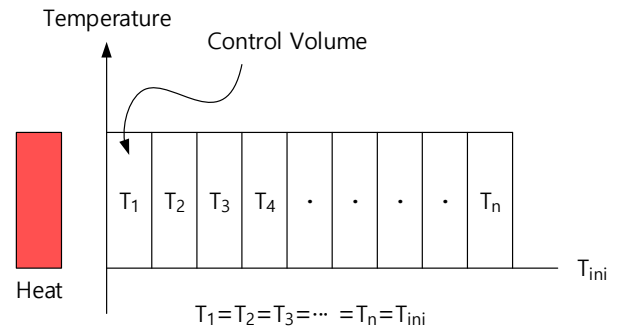
(a) Initial state at *t<0*.

(b) Transient state from *t=0* to *t=τ*.

Figure 1: Heat transfer mechanism due to a thermal disturbance. (a) An initial state, which is maintained at a constant temperature, T_{ini}, while (b) presents the transient state during *t=0* to *t=t*. At *t=0*, the temperature of the left surface of the system suddenly increased. The subsequent heat transfer rates will be reduced because the former control volume will consume a part of the heat transferred to increase its temperature, while the rest of it is transferred to neighboring shells.
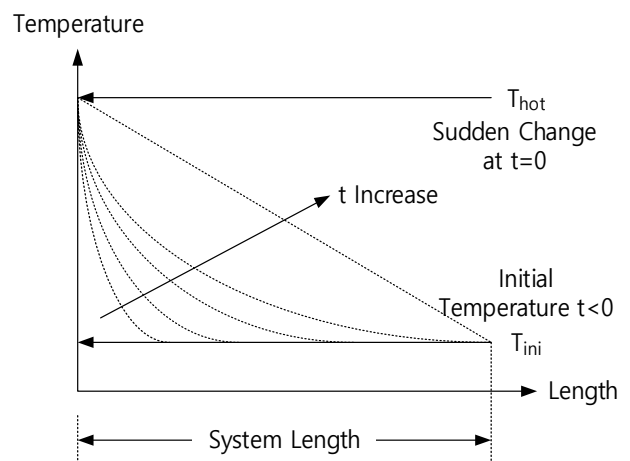

Figure 2: Predicted variation of a temperature profile with time. A sudden temperature increase is applied to the left surface of a system at *t=0*. Referring to **Figure 1 (b)**, the temperature gradient exponentially approaches a non-equilibrium steady state when the right surface is maintained at the initial temperature, T_{ini}.

However, comparing the inflow heat to the 1st shell with the inflow heat to the 2nd shell, the latter will be smaller than the former since the 1st shell consumed a part of inflow heat for the temperature rise of itself as shown in **Figure 1 (b)**. This behavior will be continued, however some shells at some distance away from the 1st shell don't perceive these thermal transient at the upstream region at all because the heat energies transferred through the shells will be gradually reduced, which means that the temperature increases in the shells will be also pro- gressively smaller.

From the physical intuition, the temperature profiles will be varied exponentially as shown in **Figure 2**. Since the heat transfer rates among the shells are consecutively reduced, each shell faces with the situation to increase its own temperature and transfer the remain heat energy to the next shell. Therefore, it is natural to pick up the exponential function for an analytical solution for **Equation (2)** although the authors had suffered from many trials and errors to find a sui- table type of function. Finding an analytical solution was started with polynomial function consisting of time and displacement, trigonometric functions, and then power series and so on.

2.2 Mathematical solution of the C-V equation

Since the temperature of a system is varied along the spatial and the time coordinates, the solution of **Equation (2)** will include the two independent variables of x and t, that is, $T=T(x,t)$. In order to incorporate the variables x and t to the exponential function as a possible solution, the form of **Equation (4)** is selected.

$$T(x,t) = Ae^{ax}e^{bt} \tag{4}$$

In **Equation (4)**, A, a and b, are the arbitrary constants at the present status. From the inspection of the proposed solution, a and b have units of m^{-1} and t^{-1}, respectively, because the exponential function will only take numeric values. This means that the constant A has to be the temperature, although the exact values of these constants are not known at this stage.

After partial differentiation of **Equation (4)** with respect to x and t, and substitution of the partial derivatives into **Equation (2)**, the following condition is obtained.

$$b^2 Ae^{ax}e^{bt} + \frac{1}{\tau}bAe^{ax}e^{bt} = C_T^2 a^2 Ae^{ax}e^{bt} \tag{5}$$

In **Equation (5)**, C_T^2 is α/τ and its square root is the thermal wave propagation speed mentioned in Section 2. Cancelling the common factor that exists in both sides of

Equation (5), **Equation (6)** is obtained.

$$b^2 + \frac{1}{\tau}b - C_T^2 a^2 = 0 \tag{6}$$

From the inspection of each term of **Equation (6)**, it is confirmed that their units are identical to s^{-2}, and maintain the dimensional homogeneity. In order for **Equation (4)** to qualify as a potential solution for the partial differential equation of **Equation (2)**, the condition of **Equation (6)** must be satisfied.

For finding the values of A, a and b, of **Equation (4)**, the two boundary conditions are applied to it: $T(x,t)=T_h$ at $t=0$ and $x=0$; $T(x,t)=T_i$ at $t=0$ and $x=l$, where l is the length of a system, as shown in **Figure 2**.

Use of the two boundary conditions, allows the determination of the two unknown constants.

$$A = T_h \tag{7}$$

$$a = \frac{1}{l}ln\left(\frac{T_i}{T_h}\right) \tag{8}$$

From **Equation (7)** and **Equation (8)**, it is found that A is simply the temperature change imposed on a boundary, and a has the units of m^{-1}. Therefore, the requirements for the appropriate units for A and a are satisfied. Since the value of b is calculated from the quadratic formula of **Equation (6)**, the two unknown values will be obtained.

$$b_1 = \frac{-\frac{1}{\tau} + \sqrt{\frac{1}{\tau^2} + 4a^2 C_T^2}}{2} \tag{9}$$

$$b_2 = \frac{-\frac{1}{\tau} - \sqrt{\frac{1}{\tau^2} + 4a^2 C_T^2}}{2} \tag{10}$$

From the inspection of **Equation (9)** and **Equation (10)**, the unit of b is still s^{-1}, which is consistent with the requirement for selecting Equation(4) as the potential solution for the C-V equation [**Equation (2)**].

Although b can take values as indicated by the two cases of **Equation (9)** and **Equation (10)**, there would be no problem to find a general solution if the superposition principle is applied to the particular solutions that have been obtained **[18]**. In order to apply the superposition principle, the general solution of **Equation (2)** will be given as follows.

$$T(x,t) = Ae^{ax}\left(e^{b_1 t} + e^{b_2 t}\right) \tag{11}$$

3. Analytical solution

Substitution of the determined constants of **Equation (7)**, **Equation (8)**, **Equation (9)**, and **Equation (10)**, into **Equation (9)**, the analytical solution of **Equation (11)** takes the following final form:

$$T(x,t) = \left[T_h e^{\left\{ \frac{x}{l} ln\left(\frac{T_i}{T_h} \right) \right\}} e^{\left(-\frac{t}{2\tau} \right)} \right] \times$$

$$\left[e^{\frac{t}{2} \sqrt{ \frac{1}{\tau^2} + \frac{4\alpha}{\tau} \left\{ \frac{1}{l} ln\left(\frac{T_i}{T_h} \right) \right\} }} + e^{-\frac{t}{2} \sqrt{ \frac{1}{\tau^2} + \frac{4\alpha}{\tau} \left\{ \frac{1}{l} ln\left(\frac{T_i}{T_h} \right) \right\} }} \right] \qquad (12)$$

In **Equation (12)**, there are four exponential terms, which are all functions of x and t. Prior to the calculation, the following functions were investigated in order to assess their characteristics.
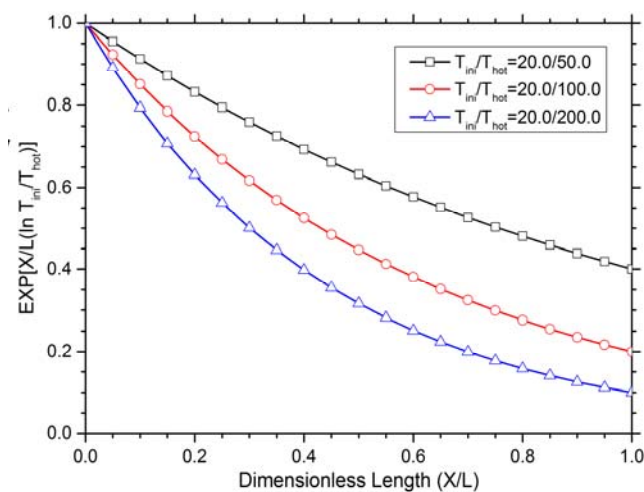
$$f_1(x) = e^{\left\{ \frac{x}{l} ln\left(\frac{T_i}{T_h} \right) \right\}} \qquad (13)$$

$$f_2(t) = e^{\left(-\frac{t}{2\tau} \right)} \qquad (14)$$

$$f_3(t) = e^{\frac{t}{\tau} \sqrt{ \frac{1}{4} + \alpha\tau \left\{ \frac{1}{l} ln\left(\frac{T_i}{T_h} \right) \right\} }} \qquad (15)$$

$$f_4(t) = e^{-\frac{t}{\tau} \sqrt{ \frac{1}{4} + \alpha\tau \left\{ \frac{1}{l} ln\left(\frac{T_i}{T_h} \right) \right\} }} \qquad (16)$$

**Figure 3 (a)** shows that the transient temperature change that occurs at the boundary progressively decreases as the distance from the boundary increases. This behavior is remarkable when the temperature difference, $\Delta T = T_h - T_i$, becomes larger. However, since the form of the exponential, spatially varying function, is continuous, it will be never become zero.

Figure 3 (b) shows the behavior of the temporally varying exponential function. One of its features is that even at a time equal to (or larger than) approximately 15 times the dimensionless time t/τ the function does not reach zero, but practically it can be treated as zero. This feature of the exponential function implies that the heat penetration depth could be approximately equal to the thermal wave speed times 15τ.

Equation (15) and **Equation (16)** can be identified only if the values of τ, α, l, T_i and T_h have to be given. Generally, the thermal diffusivities of materials have values within the range of 10^{-3} to 10^{-6} m^2/s. However, many arguments have been posed for the values of the thermal relaxation time. It is reported that it takes values in the range from 10^{-8} to 10^{-12} s, except in the case of biological tissues [19]-[21].

Figure 4 shows the variations of **Equation (15)** and **Equation (16)** with the dimensionless time, t/τ, in which the term multiplied by $t \cdot a$ in the square root was ignored because it is the constant determined by the material properties and the conditions imposed to a system. From this figure, it can be predicted that the general solution of **Equation (12)** will show an unrealistic physical situation because **Equation (15)** has the characteristic of a sharply increased function with a time. This unacceptable behavior is resulted from the shape of **Equation (15)**, which is the exponential function to be increased continuously with a time.
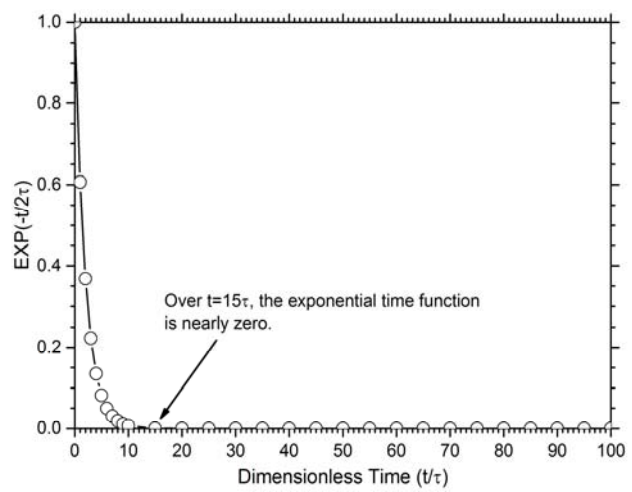
(a) Behavior of $f_1(x)$.

(b) Behavior of $f_2(t)$.

Figure 3: The behaviors of the exponential functions of **Equation (13)** and **Equation (14)** that are incorporated in the general solution of **Equation (12)** used for the solution of the C−V equation. From (b), when the time is 15 times (or more) the thermal relaxation time, t, the value of $f_2(x)$ is nearly zero. It should be noted that the value of $f_2(x)$ is not exactly zero mathematically, but practically it can be treated to be equal to zero.
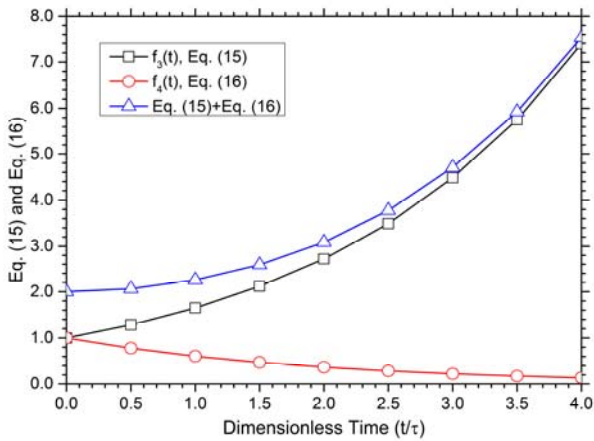
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Figure 4: The The characteristics of Equation(15) and Equation(16) included in the general solution of Equation(12). The term multiplied by ♦·☞ in the square root was ignored because it is constant.

If **Equation (12)** is accepted as the general solution without any manipulation, the temperature of the surface that is in contact with a heating or a cooling source is equal to two times the temperature of any transient source, an outcome which results from the fact that the values of **Equation (15)** and **Equation (16)** are the unit values at $t=0$, as seen in **Figure 4**.

Inspection of the behaviors of **Equation (15)** and **Equation (16)** in **Figure 4** indicates that the former increases with time, while the latter decreases to zero, and the effect disappears over 5τ. Therefore, it would be reasonable that **Equation (16)** should be excluded from the general solution of **Equation (12)** because it approaches zero with increasing time. In order to exclude the term that is not inconsistent with the actual physical phenomenon, this operation is usually applied to the procedure to find the solutions of specific differential equations **[18]**. Thus, the final form of the general solution for the non-Fourier's heat conduction of **Equation (2)** is determined as follows:

$$T(x,t) = \left[T_h e^{\left\{ \frac{x}{l} ln\left(\frac{T_i}{T_h} \right) \right\}} e^{\left(-\frac{t}{2\tau} \right)} \right] \times \left[e^{\frac{t}{2}\sqrt{\frac{1}{\tau^2} + \frac{4\alpha}{\tau}\left\{ \frac{1}{l} ln\left(\frac{T_i}{T_h} \right) \right\}}} \right] \quad (17)$$

To investigate the features of the suggested general solution for the C‑V equation, the values of α and τ should be known a priori. From the references **[19]-[21]**, the values of the thermal relaxation time in metals range from 10^{-8} to 10^{-12} s. These are extremely small values, taking into account a typical heat transfer process. If this range of

τ is inserted into **Equation (17)**, the calculated values of the exponential functions may yield tremendously large positive or negative values, which will not allow easy delineation of the features of the obtained general solution. To avoid this, the experimental data generated by K. Mitra *et al.* **[22]** are referred in this study, and they are reproduced in **Table 1**.

Table 1: Experimentally measured thermal properties of processed meat **[22]**

Material	ρ (kg/m^3)	c_p (J/kg·K)	k (W/m·K)	a (m^2/s)
Processed Meat	1230	4660	0.8	1.4*

* 10^{-7} shall be multiplied.

Note 1: Test range is about 6 oC to 29oC.

Note 2: Experimental uncertainties in the original study are not included in this table.

Note 3: Measured thermal relaxation time is distributed in 15~16 seconds.

The used values for the calculation are $\tau=15$ s and $\alpha=1.4 \times 10^{-7}$ m^2/s. The length of a specimen and the initial temperature are assumed to be 20 mm and 20 °C, respectively. Assume that beef with a thickness of 20 mm is used to cook a steak using a hot grill that is assumed to maintain the temperature of 160 °C at all times. In addition, natural convection that occurs owing to the temperature difference between the meat and the ambient temperature over its upper and side surfaces is ignored.

Figure 5 (a) shows the temperature variations in the meat that is placed on a hot grill at specific positions, which are 1 mm, 2 mm, and 3 mm from the hot surface, during the period of 5τ. Numerically, the temperature rise in the cooked meat is increased at small distances away from the hot surface. However, the changes are not. This variation results from the large value of the thermal relaxation time, and the selected positions that are too close to each other.

Figure 5 (b) shows the temperature variations with time, and the forced constraint is applied to the heated boundary. Its temperature was set at 160 °C, while the natural convection from the opposite surface was ignored. This figure clearly shows that the temperatures of the meat are increased as time progresses. Even though the extent of the temperature rise is not remarkable, the change in temperature probably resulted from the large value of the thermal relaxation time.
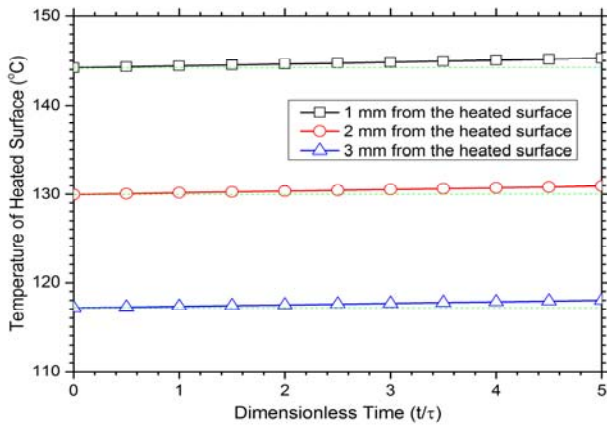
(a) Temperature variations in cooked meat as a function of position
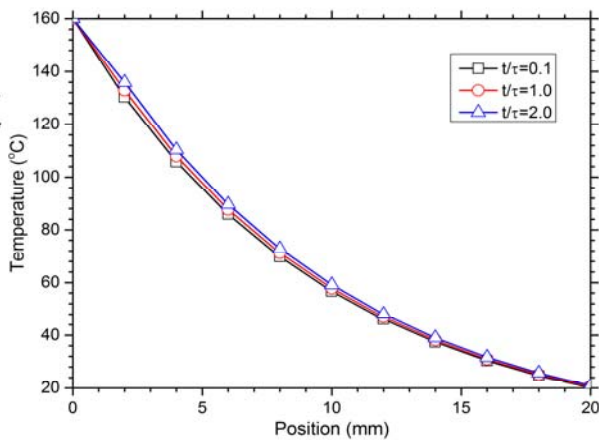
(b) Temperature variations of cooked meat as a function of time

Figure 5: Calculated temperatures in meat. The data described in Table 1 were used for calculation. In the case of (a), three spatial locations are selected from the heated boundary surface, namely 1 mm, 2 mm, and 3 mm. In the case of (b), the temporally varying temperature variations in meat are shown. Clearly, the temperatures in meat increase as the time progresses.

4. Conclusion

The purpose of the study was the identification of a simple type of a general solution for the hyperbolic partial differential equation of the non-Fourier's heat conduction, known as the C‑V equation. Up to now, many studies have suggested the use of analytical solutions for the C‑V equation. Even though consideration of this equation may accurately predict the transient state of a system, the elicited outcomes have very complicated shapes. This fact constitutes a barrier to users without extensive mathematical knowledge or skills.

The introduced concept in this study for bypassing the mathematical complication is to assume a simple solution, and confirm whether the assumed solution satisfies the originally given differential equation. Through numerous empirical tests of various functions, the simple exponential function of **Equation (4)** was confirmed to satisfy mathematically the non-Fourier differential equation of **Equation (2)**. However, the behavior of the general solution elicited unrealistic temperature profiles in the virtual experiment using published experimental data.

Although it is verified in this study that the general solution of **Equation (12)** satisfies the hyperbolic partial differential equation used for the non-Fourier's heat conduction, but is somewhat defective from a practical viewpoint. In view of these facts, the term of **Equation (16)** was excluded from the general solution since it revealed the temperature profile that is inconsistent with common sense, and therefore the final form of the general solution was determined in accordance to **Equation (17)**.

However, the authors have no assurance on whether the excluded term of **Equation (16)** is actually unrealistic or whether it contains any other meaning not perceived by the authors. Certainly, it maintains the initial temperature at the beginning of a transient state, and its effect disappears with time, which resembles the effect of thermal inertia resisting an imposed thermal disturbance.

From the calculated results, all temperatures at the various positions in cooked meat elicited higher temperatures than the initial temperature that increased with time. However, the general solution continuously increased with time.

As described in the introduction, this study is the first trial to identify a simple type of an analytical solution of the non-Fourier's heat conduction phenomena. It was performed to confirm the possibility on whether or not a simple exponential function can be used as the solution for the C‑V equation. According to this viewpoint, the results of this study provide initial evidence to the adopted approach for a simple type of a general solution for the C‑V equation.

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